PCG Convergence Analysis for Toeplitz Systems with Fisher-Hartwig Singularity

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Recently, Lu and Hurvich [2] use the PCG method with the optimal circulant preconditioner proposed by Chan [1] in 1988 for solving the Toeplitz system $T_n(f)x = b$ where the generating function f is given by

$$f(\omega) = |1 - e^{-i\omega}|^{-2d} h(\omega)$$

with $0 < d < 1/2$. The function $h(\omega)$ is positive continuous on $[-\pi, \pi]$ and differentiable on $[-\pi, \pi] \setminus \{0\}$. In this paper, we will use the superoptimal circulant preconditioner proposed by Tyrtyshnikov [3] in 1992 to solve the same problem. Our convergence analysis shows that the condition number of the preconditioned system is $\mathcal{O}(n \log^3 n)$ and therefore the complexity of our algorithm is $\mathcal{O}(n \log^4 n)$. We notice that the numerical performance of superoptimal preconditioner is almost the same as that of the optimal preconditioner. Moreover, the framework of our algorithm is more general than that established in [2].

This is a joint work with Dr. S. W. Vong and W. Wang.

References

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