Tucker Compression, Parallel Factor Analysis and Block Term Decompositions: New Results

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The two most well-known generalizations of matrix rank to higher-order tensors are multilinear rank and (outer product) rank, respectively. The computation of the best low multilinear rank approximation of a tensor is sometimes called Tucker compression. The two main applications are compression of multi-way data and the computation of dominant subspaces of a higher-order tensor. In this talk, we touch on some new algorithms and discuss the problem of local minima. The (outer product) rank of a higher-order tensor is the minimal number of rank-1 terms in which it can be decomposed. The decomposition itself is sometimes called the Canonical or Parallel Factor decomposition (CANDECOMP/PARAFAC)(CP). The most well-known condition under which essential uniqueness of the CP decomposition is guaranteed, is the famous Kruskal bound. The rank is usually determined by means of trial-and-error or by using heuristic criteria. The most popular algorithm is of the alternating least squares type. However, in many practical cases of interest, the rank is equal to the matrix rank of a matrix representation of the tensor. In the case of exact data, CP may be computed by means of standard linear algebra under certain conditions on the rank. In the case of noisy data, one can often use algorithms for simultaneous matrix diagonalization. There also exists a uniqueness condition that is significantly more relaxed than Kruskal's, provided one of the tensor dimensions is greater than its rank. The latter condition is satisfied in many applications. Recently, we have proposed Block Term Decompositions (BTD) as a unifying framework for CP and Tucker. BTD also unifies multilinear rank and (outer product) rank. We discuss BTD generalizations of our CP results.