On the Computation of Nonnegative Quadrature Weights on the Sphere and an Application to Partial Differential Equations

Manuel Gräf
Chemnitz University of Technology, Germany

In this talk we present fast algorithms for the computation of nonnegative quadrature weights of scattered sampling nodes on the two-dimensional unit-sphere. The proposed algorithms are based on fast spherical Fourier algorithms for arbitrary nodes. The obtained quadrature formulae have a high degree of exactness, i.e., they integrate up to a given accuracy all spherical polynomials of degree N. Numerical examples show that we can compute for $N \leq 1024$ nonnegative quadrature weights if approximately $4N^2/3$ well distributed nodes are used. We compare these results with theoretical statements which guarantee nonnegative quadrature weights. Furthermore we give an application from partial differential equations for the use of these quadrature formulae. There we solve efficiently a nonlinear partial differential equation which models the growth of crystals on the sphere. This so-called phase field crystal model is used to study ordering of particles on arbitrary curved surfaces. In the case of the two-sphere the solutions are related to minimal (ground state) configurations of a given number of electrons on this surface, which is also known as Thomson's problem.