Tensors and n-d Arrays: A Mathematics of Arrays (MoA) and the Psi Calculus: Composition of Tensor and Array Operations

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The Kronecker and Matrix product are key algorithms and are ubiquitous across the physical, biological, and computational social sciences. Thus, considerations of optimal implementations are important. The need to have high performance and computational reproducibility is paramount. Moreover, due to the need to compose multiple Kronecker and Matrix products, issues related to data structures and layout using an indexing algebra requires a new look at an old problem. This paper discusses the relationship between the Matrix and Kronecker Product. We discuss how both are generalized to higher dimensions and how the Inner Product is defined using the Outer Product. That is, the Inner Product is a generalization of the Matrix Product and the Outer (Tensor) Product is a generalization of the Kronecker Product. We discuss how the use of A Mathematics of Arrays (MoA), and the ψ-calculus, (a calculus of indexing with shapes), provides a normal form that is optimal, veriable, reproducible, scalable, and portable to both hardware and software. Using standard benchmarking software we have completed extensive tests on sequential and shared memory environments to demonstrate our conjectures.