Fast SVD Algorithms for Hankel Matrices

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We first consider square Hankel matrices and present an $O(n^2 \log(n))$ algorithm for computing the symmetric singular value decomposition of square Hankel matrices of order n. The algorithm consists of two stages: First, a complex square Hankel matrix is reduced to a complex symmetric tridiagonal matrix using the Lanczos method in $O(n^2 \log(n))$ flops; Second, the singular values and singular vectors of the symmetric tridiagonal matrix resulted from the first stage are computed in $O(n^2)$ flops using the twisted factorization.

Then we discuss m-by-n $(m > n)$ rectangle Hankel matrices and present a fast bidiagonalization method. An m-by-n Hankel matrix is first reduced to a real bidiagonal matrix in $O((m+n)n \log(m+n))$ flops using the Lanczos method. The singular value decomposition of the real bidiagonal matrix can then be obtained in $O(n^2)$ flops.