Preconditioned Krylov Solver For Dense Complex Systems Arising From A Fusion Application

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The presentation describes the development of a preconditioned iterative solver for large dense complex-valued linear systems arising from a fusion application for modeling the interaction of plasma and radio waves.

The next step toward fusion as a practical energy source is the design and construction of ITER (www.iter.org), a device capable of producing and controlling the high performance plasma required for self-sustaining fusion reactions, i.e. "burning plasma". Computer simulation is providing significant insight into how this can be accomplished. The AORSA [1] (All ORders Spectral Algorithm) simulation program, developed within the Scientific Discovery through Advanced Computing (SciDAC) project *Numerical Computation of Wave Plasma Interactions in Multi-dimensional Systems*, has demonstrated how electromagnetic waves can be used for driving current flow, and heating and controlling instabilities in the plasma. AORSA provides high resolution, two-dimensional solutions for mode conversion and high harmonic fast wave heating in tokamak plasma.

AORSA models the heating response of plasma due to radio frequency (RF) waves. The plasma state is described by a distribution function $f_s(\mathbf{r}, \mathbf{v}, t)$. For RF application, the fast wave time scale leads to effective approximation of the electric field, magnetic field and distribution function as a time-averaged equilibrium part ($\mathbf{E}_0$, $\mathbf{B}_0$, f_s^0) and a rapidly oscillating time-harmonic part, ($\mathbf{E}(\mathbf{r})\exp(-i\omega t)$, $\mathbf{B}(\mathbf{r})\exp(-i\omega t)$, $f_s^1(\mathbf{r}, \mathbf{v})\exp(-i\omega t)$). The time harmonic terms satisfy the generalized Helmholtz equation,

$$-\nabla \times \nabla \times \mathbf{E} + \frac{\omega^2}{c^2}\left(\mathbf{E} + \frac{i}{\omega\epsilon_0}\mathbf{J}_p\right) = -i\omega\mu_0\mathbf{J}_{antenna}$$
$$\mathbf{J}_p(\mathbf{r}, t) = \int_{-\infty}^{t} dt' \sum s \int d\mathbf{r}' \sigma(f_s^0(E), \mathbf{r}, \mathbf{r}', t, t') \cdot \mathbf{E}(\mathbf{r}', t')$$

where ω is the frequency of wave, $\mathbf{J}_p$ is the plasma current induced by the wave fields, and $\sigma(f_s^0, \mathbf{r}, \mathbf{r}', t, t')$ is the plasma conductivity kernel. Fourier modes are used as basis functions to represent the electric field. Collocation on a $M \times M$ rectangular grid is used to construct a complex linear system of size $N = 3 \times M \times M$.

A computational kernel is the construction and solution of a system of large dense complex valued linear equations. A high performance direct solver based on HPL (High Performance Linpack) has been developed to solve very large problems (N over 500,000) on 20,000 processors. The high $\mathcal{O}(N^3)$ cost for LU factorization motivates the development of an iterative solver. Due to the very large problem size and complex nature of the system, finding an effective preconditioner is a challenge. We explore the use of inverse Fourier transformation of the system into real space and consider the use of Kronecker product approximation as the preconditioner.

This Research was sponsored by the Applied Mathematical Sciences subprogram of the Office of Energy Research, U.S. Department of Energy. This research used resources of the Center for Computational Sciences at the Oak Ridge National Laboratory, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC05-00OR22725.

Reference

[1] Jaeger EF, Berry LA, D'Azevedo E, Batchelor DB, Carter MD, White KF, Weitzner H. Advances in full-wave modeling of radio frequency heated multidimensional plasmas. *Physics of Plasmas* 2002; 9(5):1873–1881.