# Preconditioners For The Incompressible Navier-Stokes Equations

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Computational Fluid Dynamics is frequently used nowadays to understand the flow in rivers, blood veins, around cars and planes, etc. This tool can also be used to make better cars and planes and to design dams and dikes to protect against flooding. In this talk we consider simulation with the incompressible Navier Stokes equations. After discretization by the Finite Element Method and linearization (using Picard or Newton-Raphson) one obtains a large sparse linear system. Due to the incompressibility constraint a large zero block appears on the main diagonal of the system matrix. This type of problems is also known as a saddle point problem. Straightforward application of direct or iterative solvers leads to breakdown or divergence of the methods. For large 3 dimensional problems only iterative methods are feasible due to large time and memory requirements of direct solvers.

For the iterative methods several Krylov subspace solvers can be used as there are: Bi-CGSTAB, GMRES, GCR and PCG. We also use a recently described Krylov solver: IDR(s). The most important part is always to find a suitable preconditioner. In the literature a number of preconditioners are given: ILU, pressure convection diffusion (PCD), least squares commutator (LSC), and augmented Lagrangian (AL). We describe some new preconditioners and compare them by numerical experiments.

First of all we present the SILU (Saddle point ILU) preconditioner. After an a priori ordering of the grid points by a Sloan reordering and a renumbering of the unknowns the standard ILU method is used. The resulting SILU preconditioner appears to be robust and fast for problems with medium sized grid sizes.

As a second preconditioner we consider an adaptation of the SIMPLE(R) method which is mostly used for Finite Volume Methods but can also be used for problems originating from Finite Elements Methods. It appears that the Modified SIMPLER (MSIMPLER) preconditioner leads to the best results comparing with the other known preconditioners. The resulting method depends only weakly on the grid size.

Finally we consider Stokes problems with variable viscosity. Applications are from geomechanics, for instance modelling the flow in the inner parts of the earth. A combination of the Schur complement method, the pressure mass matrix as preconditioner and the Multi-Level (ML) method for the subdomain problems leads to an iterative solution method which is independent of the grid size and the variation in the viscosity.

References

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