Preconditioning Hybrid Methods For Robust And Efficient Solution Of Large Highly-Indefinite Linear Systems Of Equations

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Many modern numerical simulations give rise to large sparse linear systems of equations that are becoming increasingly difficult to solve using standard techniques. Direct solvers cannot be invoked due to the infeasibly-large memory requirement. Preconditioned iterative solvers can reduce the memory requirement, but they suffer from slow solution convergence due to the ill-conditioning and highly-indefinite nature of the systems.

In order to address these challenges, researchers have been studying hybrid solvers to balance the robustness of direct solvers with the efficiency of preconditioned iterative solvers. A number of successful parallel hybrid solvers have been developed based on a domain decomposition method called the Schur complement method [4, 5]. In this method, the unknowns in the interior domains are first eliminated using the techniques from direct solvers, and the remaining Schur complement system is solved using a preconditioned iterative solver. To develop a robust and efficient parallel hybrid solver, the most critical aspect is the preconditioning technique used to solve the Schur complement system. The techniques implemented within the existing parallel hybrid solvers are often designed primarily to achieve good scalability of time to compute the preconditioners. Because of this, when these hybrid solvers are used on a large-scale computer to solve large highly-indefinite linear systems, they can suffer from either slow convergence or memory scalability, or both.

To overcome these drawbacks, we have developed a new implementation of the Schur complement method. Our hybrid solver achieves its highperformance by taking full advantages of state-of-the-art software such as the direct solvers SuperLU [3] and SuperLU_DIST [6] to form the Schur complement and the preconditioned iterative solvers from PETSc [7] to solve the Schur complement system. Furthermore, we have extended SuperLU_DIST to compute preconditioners based on parallel incomplete LU factorization. The primary advantage of our preconditioners is to provide the flexibility of balancing the memory requirement with the convergence rate. As a result, large highly-indefinite linear systems can be robustly solved on large-scale computers, and the memory requirement per processor can be reduced.

We have applied our hybrid solver to solve large highly-indefinite linear systems of equations arising from two applications, modeling next-generation particle accelerators [2] and fusion devices [1]. These applications are on the U.S. Department of Energys midterm highest-priority facility list [8]. In this talk, we will present preliminary numerical results to demonstrate the superior convergence rate achieved by our preconditioners and the robustness of our hybrid solver to solve these large highly-indefinite linear systems on a large distributed memory computer.

References

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