**Project Title**: On vector cross product on Euclidean spaces

**Supervisor**: Dr. Ho Man Ho

**Project Description**:

On $\mathbb{R}^3$ we know that vector cross product exists; i.e., for any two vectors $v$ and $w$ in $\mathbb{R}^3$, their cross product $v \times w$ is a vector in $\mathbb{R}^3$ satisfying certain properties. However, there does not seem to be a definition for cross product of two vectors in $\mathbb{R}^n$ for other $n \in \mathbb{N}$. Thus a natural question is whether $n = 3$ is the only integer for which a "meaningful" definition of vector cross product on $\mathbb{R}^n$ exists. The answer is no. Indeed, a "meaningful" definition of vector cross product on $\mathbb{R}^n$ exists if and only if $n = 0, 1, 3, 7$. There are many proofs of this fact, ranging from elementary proofs which only requires knowledge of linear/abstract algebra (classification of normed division algebras) to difficult proofs related to algebraic topology (vector fields on spheres, $H$-spaces, etc).

In this project, we will investigate several elementary proofs of the existence of vector cross product on Euclidean space and focus on the vector cross product on $\mathbb{R}^7$. Student will have a chance to learn and appreciate some topics in algebra and topology, one needs to understand some of the elementary proofs of the existence of vector cross product on Euclidean space and present it. It will be very nice if students can find new proof on the existence of vector cross product or derive the formula in a new way. References on this topic (such as textbooks and journal papers) will be provided.

Good references include (but not limited to) [3, 2, 1].

## References


1. Erik Darpö, *Vector product algebras*, Bull. Lond. Math. Soc. **41** (2009), no. 5, 898–902.
2. W. S. Massey, *Cross products of vectors in higher-dimensional Euclidean spaces*, Amer. Math. Monthly **90** (1983), no. 10, 697–701.
3. R. S. Palais, *The classification of real division algebras*, Amer. Math. Monthly **75** (1968), 366–368.