Workshop on Numerical Methods for
Hyperbolic Conservation and Balance Laws and Applications

10-11 November 2017

Hong Kong Baptist University

Special but not exclusive topics are uncertainty quantification, Multi-Level-Monte Carlo methods, measured valued solutions, statistical solutions, high performance computing.

Venue: SWT501, Shaw Tower, Shaw Campus, Hong Kong Baptist University

Speakers:

Harish Kumar, Indian Institute of Technology, Delhi

Alexander Kurganov, Southern University of Science and Technology and Tulane University

Filippo Leonardi, ETH Zurich

Kjetil Olsen Lye, ETH Zurich

De-kang Mao, Shanghai University

Keh-Ming Shyue, National Taiwan University

Jonas Šukys, Eawag: Swiss Federal Institute of Aquatic Science and Technology

Kun Xu, Hong Kong University of Science and Technology

Program

Friday, 10 Nov. 2017

9:30 - 10:00am	Welcome, Opening and Photos
10:00 - 10:45am	**Alexander Kurganov** Central-upwind schemes for shallow water models
10:45 – 11:30am	**Jonas Šukys** Uncertainty quantification using parallel multi-level Monte Carlo: applications to shallow water, Euler, magnetohydrodynamics, and multi-phase cavitation flows
11:30 - 12:00noon	Discussion
12:00 - 13:30pm	Lunch
13:30 - 14:15pm	**Kun Xu** Importance of time accurate flux function on construction of high-order schemes
14:15 - 15:00pm	**Keh-Ming Shyue** An operator splitting method for dispersive wave problems
15:00 - 15:30pm	Break
15:30 - 16:15pm	**Harish Kumar** Positivity-preserving High-order Discontinuous Galerkin Schemes for Ten-moment Gaussian Closure Equations
16:15 - 17:00pm	**Kjetil Olsen Lye** Efficient Monte-Carlo Methods for Statistical Solutions of Hyperbolic Conservation Laws
17:00 - 17:30pm	Discussion
	Dinner

Saturday, 11 Nov. 2017

9:30 - 10:15am	**De-kang Mao** Numerical dissipations in Navier-Stokes-like form for conservative front-tracking method
10:15 - 11:00am	**Filippo Leonardi** Approximating ensembles of incompressible flows
11:00 - 11:30am	Discussion and closing

Abstracts (in order of appearance)

Alexander Kurganov: Central-upwind schemes for shallow water models

Abstract: In the first part of the talk, I will describe a general framework for designing finite-volume methods (both upwind and central) for hyperbolic systems of conservation laws. I will focus on Riemann-problem-solver-free non-oscillatory central schemes and, in particular, on central-upwind schemes that belong to the class of central schemes, but has some upwind features that help to reduce the amount of numerical diffusion typically present in staggered central schemes such as, for example, the first-order Lax-Friedrichs and second-order Nessyahu-Tadmor scheme.

Jonas Šukys: Uncertainty quantification using parallel multi-level Monte Carlo: applications to shallow water, Euler, magnetohydrodynamics, and multi-phase cavitation flows

Joint work with: C. Linares, U. Rasthofer, P. Hadjidoukas, F. Wermelinger, S. Mishra, Ch. Schwab, M. Castro, and P. Koumoutsakos

Abstract: Complex liquid, gas, and plasma flow problems can be modeled in terms of nonlinear conservation laws. Examples include shallow water equations for lakes, rivers, earthquake or landslide generated tsunamis, multi-phase Euler equations for cavitating vapor cloud collapses, and Darcy's law for porous subsurface flows. Many of the above non-linear dynamical systems exhibit strong dependence on uncertain input data, such as initial data, sources and model coefficients. In this talk I will present a mathematical setting as well as two numerical methods for non-intrusive uncertainty quantification and propagation in such flows, namely finite volume method for the spatio-temporal discretization and the multi-level Monte Carlo statistical sampling technique, which accelerates standard Monte Carlo method by several orders of magnitude using clever variance reduction obtained from simulations with coarser spatio-temporal resolutions as control variates. Efficient implementation of such hierarchical discretization schemes relies on several more advanced mathematical concepts such as multi-level alias-free representation for random bathymetry and unbiased spectral parallel FFT generation of random porosity fields. Numerical experiments using in-house developed ALSVID-UQ, Cubism-MPCF, and PyMLMC software up to one trillion mesh elements and 500'000 cores will be presented, illustrating the efficiency of the methods and pushing be boundaries of current scientific knowledge in this field.

Kun Xu: Importance of time accurate flux function on construction of high-order schemes

Abstract: The higher order CFD methods for compressible flow are mostly based on WENO and DG formulations, where the exact or approximate Riemann solver is used for the flux evaluation. The use of the 1st-order Riemann flux function may be the barrier for the further development of higher-order accurate, robust, and efficient methods. In CFD community, the necessity of using high-order flux function, such as those based on the generalized Riemann problem and gas-kinetic scheme, has not been fully recognized. In this talk, we are going to demonstrate the importance of high-order time accurate flux function, and its usage in the

development of higher-order schemes. With the support of numerical examples, the superior advantages of the newly developed higher-order gas kinetic schemes over the existing WENO and DG methods will be presented.

Keh-Ming Shyue: An operator splitting method for dispersive wave problems

Abstract: Our aim in this talk is to describe a simple operator-splitting approach for the efficient numerical simulation of dispersive wave problems. The algorithm uses the Iordanski-Kogarko-Wijngaarden model (cf. [1, 2, 4]) for pressure waves in bubbly liquids as the basis, and reformuate it into a hyperbolic-elliptic system so that higher-order derivatives terms modelling dispersive effects of solutions can be handled straightforwardly by the method. Sample numerical results are shown to demonstrate the feasibility of the proposed method for a class of benchmark problems in bubbly liquids, and also for problems modeled by the Green-Naghdi equations of the long-wave shallow water flow [3].

References

[1] S.V. Iordanski. On the equations of motion of the liquid containing gas bubbles. Z. Prik. Mekh. Tekhn. Fiziki, N3:102-111, 1960 (in Russian).
[2] B.S. Kogarko. On the model of cavitating liquid. Dokl. AN SSSR, 137:1331-1333, 1961 (in Russian).
[3] O. Le Metayer, S. L. Gavrilyuk, and S. Hank. A numerical scheme for the Green-Naghdi model. J. Comput. Phys., 229:2034-2045, 2010.
[4] L. van Wijngaarden. On the equations of motion for mixtures of liquid and gas bubbles. J. Fluid Mech., 33:465-474, 1968.

Harish Kumar: Positivity-preserving High-order Discontinuous Galerkin Schemes for Ten-moment Gaussian Closure Equations

Joint work with: Dr. Praveen Chandrashekar (TIFR-CAM, Bangalore) and Ms. Asha Meena (IIT Delhi)

Abstract: Euler equations for compressible flows treats pressure as a scalar quantity. However, for several applications this description of pressure is not suitable. Many extended model based on the higher moments of Boltzmann equations are considered to overcome this issue. One such model is Ten-moment Gaussian closure equations, which treats pressure as symmetric tensor.

In this work, we develop a higher-order, positivity preserving Discontinuous Galerkin (DG) scheme for Ten-moment Gaussian closure equations. The key challenge is to preserve positivity of density and symmetric pressure tensor. This is achieved by constructing a positivity limiter. In addition to preserve positivity, the scheme also ensures the accuracy of the approximation for smooth solutions. The theoretical results are then verified using several numerical experiments.

In the second part of the talk, I will discuss how central-upwind schemes can be extended to hyperbolic systems of balance laws, such as the Saint-Venant system and related shallow water models. The main difficulty in this extension is preserving a delicate balance between the flux and source terms. This is especially important in many practical situations, in which the solutions to be captured are (relatively) small perturbations of steady-state solutions. The other crucial point is preserving positivity of the computed water depth (and/or other quantities, which are supposed to remain nonnegative). I will present a general approach of designing well-balanced positivity preserving central-upwind schemes and illustrate their performance on a number of shallow water models.

Kjetil Olsen Lye: Efficient Monte-Carlo Methods for Statistical Solutions of Hyperbolic Conservation Laws

Joint work with: Siddhartha Mishra and Ulrik Fjordholm

Abstract: An open question in the field of hyperbolic conservation laws is the question of well-posedness. Recent theoretical and numerical evidence have indicated that multidimensional systems of hyperbolic conservation laws exhibit random behavior, even with deterministic initial data. We use the framework of statistical solutions to model this inherit randomness. We review the theory of statistical solutions for conservation laws.

Afterwards, we introduce a convergent numerical method for computing the statistical solution of conservation laws, and prove that it converges in the Wasserstein distance through narrow convergence for the case of scalar conservation laws. For the scalar case, we validate our theory by computing the structure functions of the Burgers' equation with random initial data. We especially focus on Brownian initial data, and the measurement of the scalings of the structure functions. The results agree well with the theory, and we get the expected convergence rate. We furthermore show that we can get faster computations using Multilevel Monte-Carlo for computing the statistical solutions of scalar conservation laws.

In the case of systems of equations, we test our theory against the compressible Euler equations in two space dimensions. We check our numerical algorithm against two ill-behaved initial data, the Kelvin-Helmholtz instability and the Richtmeyer-Meshkov instability, and compute the corresponding structure functions. We furthermore show that in the case of these ill-behaved initial data, Multilevel Monte-Carlo cannot improve upon the Monte-Carlo algorithm in computing the statistical solutions.

De-kang Mao: Numerical dissipations in Navier-Stokes-like form for conservative front-tracking method

Abstract: In simulations of interfacial instabilities Euler system of fluid dynamics is taken as the governing equations, and in doing so the small-scale physical dissipations, mass diffusion, viscosity and heat condition, are ignored. In capturing simulations, small-scale roll-ups on the interfaces occur in early times and are believed to be the artifacts caused by numerical

dissipations of the capturing methods. Front-tracking simulations eliminate the numerical dissipations across the interfaces; however, they lack proper mechanism to stabilize the interface and the situation will get worse. This talk presents numerical dissipations designed for a previously developed conservative front-tracking method, which stabilize the tracked interfaces and prevent the early-time roll-ups. These numerical dissipations simulate in someway the missing physical dissipations in the Euler system on the interfaces. The designed numerical dissipations are of the order of mesh size; therefore, they tend to zero along with the grid refinement. Numerical examples are presented to show the efficiency and effectiveness of these numerical dissipations.

Filippo Leonardi: Approximating ensembles of incompressible flows

Abstract: Ensembles of solutions arise when considering the flow as a statistical quantity rather than a deterministic one. The main benefits of considering ensembles of solutions are two. On one hand, it allows us to obtain (statistical and "in the large") information about the fluid behaviour, information that would otherwise be impossible to gather by looking at the individual solution. On the other hand, it allows us to incorporate the notion of uncertainty quantification within the same framework, at no extra cost.

We will discuss two frameworks for ensembles of solutions: statistical solutions, i.e. solutions as push-forward measures on function spaces, and measure valued solutions, i.e. space-time parametrised Young measures. We will focus on the efficient approximation of these types of solutions and discuss implementations of those approximations.