On the convergence of the parallel non-stationary multisplitting iteration methods

Zhong-Zhi Bai

Institute of Computional Mathematics and Scientific/Engineering Computing, Chinese Academy of Sciences

The convergence properties of the non-stationary multisplitting iteration method, a variant of the parallel chaotic multisplitting iteration method for solving the large sparse system of linear equations presented by Bru, Elsner and Neumann (Linear Algebra and Its Applications, 103(1988)175-194), are further discussed when the coefficient matrix is an H-matrix and a positive definite matrix, respectively. Moreover, when the coefficient matrix is a monotone matrix, the monotone convergence theory and the monotone comparison theorem about this method are established. This directly leads to several novel sufficient conditions for guaranteeing the convergence of this parallel non-stationary multisplitting iteration method.

The random projection method for hyperbolic conservation laws with stiff source terms

Weizhu Bao

Tsinghua University & Georgia Tech

Shi Jin

School of Mathematics Georgia Institute of Technology

In this paper we propose the random projection method for numerical simulations of the hyperbolic conservation law with stiff source term arising from chemically reactive flows:

$$U_t + F(U)_x + G(U)_y = \frac{1}{\varepsilon}\Psi(U).$$

In this problem, the chemical time scales may be orders of magnitude faster than the fluid dynamical time scales, make the problem numerically stiff if one uses an underresolved numerical method which resolves only the fluid dynamics scales. A classic spurious numerical phenomenon, the incorrect propagation speeds of discontinuities, occur in underresolved numerical solutions. We introduce a random projection method for the reacting term by replacing the ignition temperature by a uniformly distributed random variable. The statistical average of this method corrects the spurious shock speed, as will be proved with a scalar model problem and demonstrated by a wide range of numerical exmples in denotation waves.

On the Nonlinear Inexact Uzawa Algorithm for Saddle Point Problem

Xiao-liang Cheng
Department of Mathematics, Hangzhou University

Convergence behavior of the nonlinear inexact Uzawa algorithm for solving saddle point problems presented in a recent paper of J.H. Bramble, J.E. Passciak and A.T. Vassilev (SIAM J. Numer. Anal., vol. 34(1997), 1072-1092) is discussed. It is shown that this algorithm converges under conditions weaker than that stated in their paper.

Wavelets On Interval Over Regular or Irregular Meshes

I-Liang Chern and Chien-Chang Yen
Department of Mathematics, National Taiwan University

We construct compact supported biorthogonal wavelets on intervals with underlying meshes being uniform or nonuniform. At the first step, a sequence of fundamentals are constructed based on the Lagrange interpolation through a subdivision scheme. These fundamentals are interpolating and supported inside the interval. The boundary fundamentals are constructed based on one-side interpolation instead. At the second step, two multi-resolution analyses are constructed. Its primals consist of B-splines, while the duals are the summations of derivatives of the above fundamentals. The biorthogonality of these two multi-resolution are followed by a commutation formula. A pair of biorthogonal wavelets are thus constructed. Riesz basis property is investigated. Regularity of these wavelets over irregular meshes is also studied for the case when the above Lagrange interpolation is cubic.

A method to generate quadrature rules for oscillatory integrals

Kwok-Chiu Chung

Dept of Applied Mathematics, Hong Kong Polytechnic University

In a recent paper, Evans and Webster produced a quadrature rule for oscillatory integrals with a trigonometric kernel. Their method is a modification of an earlier method due to Levin, and involves making a quadrature rule exact for a set of functions for which the modified moments have a simple closed form. This approach is now extended to deal with other forms of kernel, including Bessel functions and combinations of Bessel and trigonometric functions. In fact, the extension is suitable for any oscillatory kernel provided only that it satisfies a known linear ordinary differential equation. There is some freedom in the choice of differential equations used and this can sometimes be used to generate a number of different quadrature rules.

The Two-Scale Analysis Method for the Structure of Composite Materials

Jun-Zhi Cui

Institute of Computional Mathematics and Scientific/Engineering Computing, Chinese Academy of Sciences

Since composite material has periodicity in configuration the structures made from composite materials is considered as the structures with small periodic configurations. The mechanical behaviors of this kind of structures do not only depend on the macroscopic characteristics, such as the geometry of structure, effective material constants, the loading and the constraints, but also on detailed configuration in basic cell, such as the matrix, the reinforce and their interfaces. The stress analysis for the structures leads to solving the boundary value problem of Lame equations with the coefficients of small period. The Two-Scale Asymptotic analysis (TSA) and its FE analysis have been developed for first boundary value problem in 2 and 3 dimensional cases by author. The TSA and its FE computation of mixed boundary value problem for a class of composite structures are discussed in this paper.

1. The displacement solution can be expressed in TSA as follows

$$\mathbf{u}(\mathbf{x}) = \begin{cases} \mathbf{u}_0^1(\mathbf{x}) + \sum_{l=1}^{\infty} \varepsilon^l \sum_{\alpha \in V_l} \mathbf{N}_\alpha(\xi) \dfrac{\partial^l \mathbf{u}_0^1(\mathbf{x})}{\partial x_{\alpha_1} \cdots \partial x_{\alpha_l}}, & \mathbf{x} \in \Omega_1 \\ \mathbf{u}_0^2(\mathbf{x}), & \mathbf{x} \in \Omega_2 \end{cases}$$

where

$$\mathbf{u}_0(\mathbf{x}) = \begin{cases} \mathbf{u}_0^1(\mathbf{x})\, \mathbf{x} \in \Omega_1 \\ \mathbf{u}_0^2(\mathbf{x})\, \mathbf{x} \in \Omega_2 \end{cases}$$

is homogenized solution on $\Omega = \Omega_1 \cup \Omega_2$, every $\mathbf{N}_\alpha(\xi) = (\mathbf{N}_{\alpha 1}(\xi), \cdots, \mathbf{N}_{\alpha n}(\xi))$ is a 1-periodic function matrix and $\mathbf{N}_{\alpha m}(\xi)$ is a vector-valued solution on unit-square basic configuration .

2.The algorithm procedure based on TSA is given. The FE solution $\mathbf{N}_{\alpha m}^{h_1}(\xi)$ of periodic solutions is computed in a unit-square similar to basic configuration, and its error is given. Then the approximate macroscopic constants $\{a_{ijhk}^h\}$ and homogenized FE solution $\mathbf{u}_0^{h_0}(\mathbf{x})$ in whole domain Ω are computed and the error is estimated.

3. The FE formulas of the displacements based on TSA are shown

$$\mathbf{u}^{hm}(\mathbf{x}) = \begin{cases} \mathbf{u}_0^{1h_0}(\mathbf{x}) + \sum_{l=1}^{m} \varepsilon^l \sum_{\alpha \in V_l} \mathbf{N}_\alpha^{h_1}(\xi) \dfrac{\partial^l \mathbf{u}_0^{1h_0}(\mathbf{x})}{\partial x_{\alpha_1} \cdots \partial x_{\alpha_l}}, & \mathbf{x} \in \Omega_1 \\ \mathbf{u}_0^{2h_0}(\mathbf{x}), & \mathbf{x} \in \Omega_2 \end{cases}$$

and then stress formulas are given.

4. The numerical results for a 2-D typical problem are given in three models of basic configuration. The results show the efficiency and precision of FE algorithms based on TSA.

Landau-Lifshitz equation and ferromagnetics: analytical and physical issues

Weinan E
Courant Institute, NYU

This talk is an introduction to the subject of domain structures and domain walls in ferromagnetic materials. Both statics and dynamics will be discussed. The present talk concentrates on the analytical issues such as well-posedness of the PDE, various asymptotic limits, domain theory, etc. We will also discuss the physical background of these issues. My talk for the meeting in Beijing will concentrate on the numerical issues.

Interior Point Methods for Semidefinite Programming

Ming Gu
Department of Mathematics, UCLA

A number of primal-dual interior point methods for the semidefinite programming problem (SDP) were developed in recent years. But their implementation had lead to very complex computational behaviour. We analyze these methods in finite precision arithmetic and explain why these methods behaved the way they did. Based on our analysis, more efficient and reliable methods have now been developed.

Numerical Analysis of Variational Inequalities Arising in Contact Problems

Weimin Han

Department of Mathematics, University of Iowa

In this talk we give a summarizing account of numerical analysis of several classes of variational inequalities arising in contact problems. The materials involved are viscoelastic or viscoplastic, and the contact can be frictional or frictionless. The talk is based on my joint work with Mircea Sofonea of the University of Perpignan.

Penalty methods for optimal boundary control problems

Steve Hou

Dept of Mathematics, Iowa State University

Penalty methods for solving optimal boundary control problems for the Navier-Stokes equations are proposed and analyzed mathematically and numerically. Convergence of the penalty solutions is proved, error estimates are derived, and computational results are presented.

A Generalized Finite Element Method for Elliptic Problems with Highly Oscillating Coefficients

Yunqing Huang
Dept of Mathematics, Xiangtan University
Jinchao Xu
Dept of Mathematics, Penn State

In this talk, a generalized finite element method is presented for solving elliptic problems with highly oscillating coefficients. Some sharp convergence estimates are given for the periodic structures which contains two (slow and fast) scales by employing the homogenization theory while our method is suited for multiscale problems even for problems with non-periodic structures.

Regularization of the Burnett Equations via Relaxation

Shi Jin
School of Mathematics, Georgia Tech

The classical Chapman-Enskog expansion on the Boltzmann equation yields, to third order, linearly unstable Burnett equations that are inconsistent to the second law of thermodynamics. Past work using rational approximations to fix this instability yields nonlocal systems that are impractical numerically. We introduce a viscoelastic relaxaion system that are local, hyperbolic, endowed with a generalized Clausius-Duhrem inequality (thus the second law of thermodynamics), and agrees with the Boltzmann equation up to the Burnett order. This offers a new way to close the Boltzmann equations using thirteen "moments", and sheds lights on possible simulation of rarefied gas dynamics and fluids with an accuracy beyond the Navier-Stokes order. This is a joint work with Marshall Slemrod.

Generalized Time-Invariant Noise Algorithm and Related Bifurcation Problem

Gongyan Lei

School of Mathematics, Peking University

By introducing a parameter α, the Time-Invariant Noise Algorithm (TINA) applying to global optimization problems is generalized to a family of algorithms, it is demonstrated that for $\alpha > 0.5$, the solution of generalized TINA converges to a global minimum in probability; However, for $\alpha \leq 0.5$, the solution depends on both the function being considered and the space dimension, that is, if the process of searching for a solution is considered as a stochastic dynamical system, then, there will be bifurcation phenomena and both the α and the space dimensions will be the branch parameters. The meaning of above results is discussed and the numerical tests support our theory.

Introduction to the Immersed Interface Method and recent Progress

Zhilin Li

Center For Research in Scientific Computation and Department of Mathematics, North Carolina State University

Many physical problems involve interfaces. Examples include phase transition problems where the interface separates the solid and liquid regions, bubble simulation, Hele-Shaw flow, composite materials, and many other important physical phenomena. Mathematically, interface problems usually lead to differential equations whose input data and solutions have discontinuities or non-smoothness across interfaces. As a result, many standard numerical schemes do not work or work poorly for interface problems.

In the Immersed Interface Method, we use Cartesian grids and modify finite difference methods instead of using adaptive or body fitting grids to resolve the solutions near or on interface. In this talk, I will briefly explain how the Immersed Interface Method is used to solve certain partial differential equations; the advantages and problems associated with the method; recently advances in this area, particularly, the finite element version of the method which guarantees the stability and convergence of the method; and applications if time permits.

Non-oscillatory High-resolution Central Schemes for Hamilton-Jacobi Equations

Chi-Tien Lin

Department of Applied Mathematics, Providence University, Taiwan

In this talk, we construct second-order central schemes for multi-dimensional Hamilton-Jacobi equations and we show that they are non-oscillatory in the sense of satisfying the maximum principle. Thus, these schemes provide the first examples of non-oscillatory second-order Godunov-type schemes based on *global* projection operator. Numerical experiments are performed; L^1/L^∞-errors, and convergence rates are calculated. For convex Hamiltonians, numerical evidence confirms that our central schemes converge with second-order rate, when measured in the L^1-norm advocated in our recent result regarding L^1-werror estimate. The standard L^∞-norm, however, fails to detect this second-order rate.

Finite Element Methods For Lower Undergrades

Qun Lin

Institute of System Sciences, The Chinese Academy of Science

Many years ago Professor Feng Kang told me that the first finite element example was given by Euler. The following is my understanding. Before Euler, there was the foundamental calculus formula invented by Newton. A modification for such a formula leads to the Euler algorithm, which gives a piecewise linear solution for a differential equation and so each can be regarded as a finite element example. So, we can talk with lower undergrades about finite element methods when talk about fundamental calculus formula. I suppose this is a goal of education reform to talk with lower undergrades with some new tools in this century.

Analysis and Computation of 2-D Discontinuous incompressible Euler Flows

Jian-Guo Liu

Institute for Physical Science and Technology and Department of Mathematics, University of Maryland

Numerical simulation of 2-D discontinuous incompressible flows is of considerable interests in both theoretical analysis and applications. For inviscid inter-facial flows, it is believed that the Lagrangian methods such as vortex methods give preferable treatments. However, the convergence of such methods poses great difficulties due to the Kevin-Helmholtz instability, or ill-posedness in mathematical content. We will show both vortex blob methods and point vortex methods are indeed converge.

For more complicated flows such as a flow mixing grid-based methods such as finite difference and finite elements are commonly used. We will present a discontinuous Galerkin method for two dimensional incompressible flow in vorticity streamfunction formulation. In this method, the momentum equation is treated explicitly, utilizing the efficiency of the discontinuous Galerkin method. The streamfunction is obtained by a standard Poisson solver using continuous finite elements. There is a natural matching between these two finite element spaces, since the normal component of the velocity field is continuous across element boundaries. This allows for a correct upwinding gluing in the discontinuous Galerkin framework, while still maintaining total energy conservation with no numerical dissipation and total enstrophy stability. The method is suitable for inviscid or high Reynolds number flows. On the theoretical side, we are able to show that the numerical solutions converge strongly both in space and time for a class of discontinuous initial data $\omega_0 \in L^2$. On the numerical side, a variety of discontinuous flows such as strong shear flow and vortex patches has been carefully resolved with the method. This is a joint work with Chi-Wang Shu and Zhouping Xin.

Finite Element Approximation of Some Degenerate Nonlinear Systems

Wen-Bin Liu and N. N. Yan
Institute of Mathematics & Statistics, Univ. of Kent, England

In this work we present an overview of some recent research on finite element approximation of degenerate nonlinear systems.

We first discuss some of the unique difficulties in a priori error analysis for finite element approximation of such systems. A key step in the analysis is to introduce system related norms to handle degenerate natures of the systems.

We then discuss some issues in numerical computations of the systems, including some initial theoretical and numerical results in adaptive finite element approximation of the p-Laplacian. Some results on domain decomposition and multigrid methods are also discussed.

A New Boundary Condition Capturing Method for Elliptic Interface Problems

Xu-Dong Liu Thomas Sideris
Dept of Mathematics, UCSB
Ronald P. Fedkiw and Myungjoo Kang
Dept of Mathematics, UCLA

Interfaces have a variety of boundary conditions (or jump conditions) which need to be enforced. While a natural tendency in the literature is to explicitly enforce these interface boundary conditions, we instead propose a new boundary condition capturing technique. Since we capture the boundary conditions as opposed to explicitly enforcing them, our method is robust and easy to implement even in three spatial dimensions.

The Ghost Fluid Method (GFM) was developed to capture boundary conditions at a contact discontinuity in the inviscid Euler equations. This method was extended to treat more general discontinuities such as shocks, detonations, and deflagrations and compressible viscous flows. In this paper, we take a similar boundary condition capturing approach to the boundary conditions associated with the Poisson equation.

We also formulate the interface problem in weak form and prove existence, uniqueness and regularity of the solution. Our numerical scheme satisfies the discrete version of this weak

Regularity and Stability for an Inverse Scattering Problem

Gang Bao
Department of Mathematics, University of Florida
Yu Chen
Courant Institute, NYU
Fuming Ma
Department of Mathematics, Jilin University

The main goal of this paper is to study the linearization of an inverse medium problem. Regularity and stability results are established for the near-field *scattering map* (or scattering matrix) which maps the scatterer to the scattered field. Properties on continuity and Fréchet differentiability of the scattering map are examined. A lower bound of the linearized scattering map is obtained, which in particular characterizes the stability properties. These results are crucial for linearization of the nonlinear scattering map.

A Spectral-Petrov-Galerkin Method for Third-Order Differential Equations

Heping Ma
Department of Mathematics, Shanghai University
Weiwei Sun
Department of Mathematics City University of Hong Kong

Spectral methods are efficient for solving second- and fourth-order equations. However, it seems that we still don't know well the behavior of spectral methods for third-order equations. We analyse the Legendre spectral approximation to the third-order equation and give error estimates of the method. Also, we propose a Legendre-Petrov-Galerkin method for the problem. We show that this new approach is more reasonable than the standard one. It is easier to be implemented and the optimal error estimate in L^2-norm can be obtaind. The method is then applied to the third- and fifth-order Korteweg-de Vries equations and their generalizations with the Chebyshev collocation treatment of the nonlinear term: A Chebyshev-Legendre method.

To illustrate the application of this new approach, we give numerical results for the Korteweg-de Vries equation with a single soliton solution and the collision of two solitons.

V-cycle Multigrid Methods for TRUNC Plate Element

Zhong-Ci Shi and Xuejun Xu
Institute of Computional Mathematics and Scientific/Engineering Computing, Chinese Academy of Sciences

We present two V-cycle multigrid methods for the nonconforming TRUNC plate element. The first one uses the same nonconforming finite element space for the both coarse and fine grids. It is shown that the method is convergent and requires only one smoothing step on each level.

The second method uses the Powell-Sabin conforming finite element space as the coarse grid correction space. The uniform convergence rate independent of mesh sizes and levels is established. The method requirs only one smoothing step on all coarse levels, while on the last level a sufficient number of smoothing steps is still needed.

Weighted Essentially Non-Oscillatory Schemes on Triangular Meshes

Chi-Wang Shu
Division of Applied Mathematics, Brown University

In this talk we present a class of third order and fourth order accurate weighted essentially non-oscillatory (WENO) schemes on two dimensional unstructured meshes (triangles) in the finite volume formulation. The idea is to write a higher order reconstruction as a linear combination of lower order reconstructions with smaller stencils, and then introduce nonlinear weights into such a combination so that accuracy in smooth regions and non-oscillatory property near shocks can be obtained simultaneously. We present a new method to handle negative weights, which appear when the triangulation is highly non-uniform. Numerical examples will be shown. This is joint work with Changqing Hu.

An Artificial Boundary Condition for Vortex in Cell Method

Zhen-huan Teng
School of Mathematics, Peking University

We adapt some of the artificial boundary conditions for Poisson equation to the vortex method for imcompressible Euler flow and presnt several numerical experiments to show its effects.

Robust Multigrid Methods Using Energy-Minimizing Interpolation and Interface Preserving Coarsening

Justin Wan
Scientific Computing, Stanford University

In this talk, we describe two new approaches for designing robust multigrid methods. The construction of a robust interpolation operator is based on the concepts of energy minimization and approximation. It can handle PDE coefficients of various types on structured or unstructured grids under one framework. The formulation is general; it can be applied to any dimension. The theoretical analysis for one dimension proves that the convergence rate of the resulting multigrid method is independent of the PDE coefficients and of the mesh size. We demonstrate numerically the effectiveness of the multigrid method in two dimensions by applying it to a discontinuous coefficient problem, an oscillatory coefficient problem and an anisotropic problem. Empirically, the convergence rate is also independent of the PDE coefficients, in addition to being independent of the mesh size. The proposed method is primarily designed for second order elliptic PDEs, with possible extensions to other classes of problems such as integral equations.

The interface preserving coarsening is a geometric technique specifically for solving discontinuous coefficient PDEs. It selects coarse grid points so that all the coarse grids are aligned with the interfaces for regular interface problems on structured grids, and that the interfaces are resolved as much as possible for irregular interface problems. As a result, multigrid with linear interpolation is sufficient to obtain fast convergence. We show by interface problems in one and two dimensions that the convergence rate of the resulting multigrid method is independent of the mesh size and the size of the jump at the interfaces.

Mathematical Theory and Numerical Methods in Option Pricing

Junping Wang

Department of Mathematics, Wyoming University

In this talk we will introduce a new formulation for the valuation of American options by using a weak variational form for the corresponding free boundary value problem derived by Robert Merton. The weak form is formulated for the time-value of the American call option. It is proved that the weak form has a unique solution, and the optimal exercise curve is given as the interface of positive time-values from zero time-values. The new formula for the valuation of American options is advantageous over the classical one in that the identification of the optimal exercise curve (free boundary) is separated from the determination of the valuation function. Based on the new variational form, some innovative numerical schemes can be designed to provide very accurate approximate solutions for the American options.

An existence of weak solutions for the valuation of American options is established as the limit of a sequence of approximate solutions obtained from discretizing the continuous problem in time by the backward Euler method. Some new mathematical techniques are developed in order to deal with the discontinuous nonlinearity in the weak form due to the use of the Heaviside function. The proof of the existence result also provides a convergent numerical scheme for approximate solutions of American options. The Schwarz overlapping domain decomposition method will be applied to the discretized nonlinear problem.

On Computational Geometry

Ren-Hong Wang

Dept of Mathematics, Dalian University of Technology

In this talk, we will talk about some recent results on computational geometry. Moreover some open problems will be also introduced.

Numerical computation of the incompressible MHD equation

Weicheng Wang
Department of Mathematics, National Tsinghua University, Taiwan

We present some efficient finite difference methods for the incompressible MHD equation. This is joint work with Professor Jian-Guo Liu at University of Maryland.

On the Invariant to Determine the Convergence of Iterations and Universal Constant

Xinghua Wang
Department of Mathematics and Information Science, Hangzhou Unversity

The problem to solve the operator equation $f(x) = 0$ in a abstract space is a generalized applied mathematical subject. There are many literatures which study the existence and uniqueness of the solution of the equation in the neighbourhood of an initial x_0 and discuss the convergence of the sequence $\{x_n\}$ which is produced by an iteration with the initial x_0. The iterations have lots of different forms, for example, Newton's iteration is

$$x_{n+1} = x_n + \delta_f(x_n) := x_n - f'(x_n)^{-1} f(x_n), \quad n = 0, 1, \cdots .$$

Is it possible to unify the determinations for these different forms of iterations?

In this paper, we prove that the criterion $\beta = \|\delta_f(x_0)\|$ which was introduced by Smale(1986) for studying the convergence of Newton's method is an invariant to determine the convergence of many iterations which take Newton's step $x_0 + \delta_f(x_0)$ as the first step or substep. We give a value $b = r - \int_0^r \omega(u)du$ for certain natural subclass of operator class satisfying the condition

$$\|f'(x_0)^{-1} f'(x) - I\| \leq \omega(\|x - x_0\|),$$

which is a universal constant to determine the existence and uniquess of the solution and the convergence of many iterations.

An iterative grid redistribution method for singular problems in multiple dimensions

Xiao-Ping Wang
Department of Mathematics, HKUST

For the numerical solution of time dependent PDE's that involve large solution variations, a variety of grid redistribution methods based on equidistribution (in one dimension) or a variational approach (in multiple dimensions) have been shown to gain improvements in accuracy and efficiency over the conventional fixed mesh method. However, direct applications of many of those methods to PDE's with singular behaviors (e.g. finite time blow up) often fail due to the lack of control of the resulting mesh distribution. We introduce a new iterative grid redistribution method to solve the singular solutions. Our method requires little prior information of the singular solutions and can handle multiple singularities.

Computational Modelling for Hydrodynamics Equations using Meshless Radial Basis Functions

S.M. Wong and T.S. Li
Open University of Hong Kong, School of Science & Technology
Y.C. Hon
City University of Hong Kong, Mathematics Department

This paper introduces the meshless radial basis function methods for computing the numerical solution of partial differential equations. Enhancement to the computational efficiency and improvement of the conditioning of the full coefficient matrix can be achieved by using compactly supported radial basis functions. In this way the system of the full matrix is reduced to a banded matrix. Compactly supported scheme leads to an efficient algorithm for the computation of the interpolants. The application of the methods in solving a set of non-linear time-dependent hydrodynamics equations is presented. The proposed model is applied specifically to simulate the variations of tidal current and water flow circulation pattern for marine environments by taking considerations of the topographical and wind conditions. The numerical solutions are verified with the observed data which are collected at a specific period. The model exhibits agreement between the simulated and the observed data.

Convergence of A Galerkin Method for 2-D Discontinuous Euler Flows

Zhouping Xin
The Institute of Mathematical Sciences, CUHK and Courant Institute, NYU

In this talk, I will present a convergence result for a discontinuous Galerkin method which yields approximate solutions to the 2-D imcompincompressible Euler equations with initial vorticity which are square summable. This is proved as a consequence of the conservation of energy, boundness of the enstropy, and higher order time regularity of the method. In the case that the initial vorticity is bounded and measuarble, the whole sequence is shown to be strongly convergent. The theory applies in particular to the important class of physical flows: vortex patches.

Some recent developments in multigrid methods

Jinchao Xu
Department of Mathematics, Penn State

This talk will report a number of new algorithms and new theories on multigrid methods that the speaker has developed recently with his collaoborators.

The Generalization of Brezzi's Theory on Hybrid-Mixed Finite Element Method

Weimin Xue
Department of Mathematics, Hong Kong Baptist University

In this talk the general hybrid-mixed finite element methods are investigated systematicall in a framewark of multi-field variational formulation. The commonly accepted concept "saddle point problem" is argued in this talk. The existence, uniqueness, convergence and stability properties of the finite element solution are proved undertaking the assumptions of Ker*-ellipticity and nested BB- conditions. The relation between discrete BB-condition, rank condition and smallest singular value are discussed.

Residual Replacement Strategies for Krylov Subspace Iterative Methods for the Convergence of True Residuals

Henk A. van der Vorst
Department of Mathematics, Utrecht University, The Netherlands
Qiang Ye
Department of Mathematics, University of Manitoba

In this work, we consider maintaining convergence of true residuals in finite precision implementation of Krylov subspace methods for solving $Ax = b$. A strategy is proposed for alternative computations of the residual vectors, which improves the agreement of the computed residuals and the true residuals to the level of $O(\mathbf{u})\|A\|\|x\|$, where $\mathbf{u}$ is the roundoff unit. Building on earlier ideas on residual replacement and on insights in the finite precision behaviour of the Krylov subspace methods, computable error bounds are derived for iterations that involve occasionally replacing the computed residuals by the true residuals, and they are used to monitor the deviation of the two residuals and hence to select residual replacement steps, so that the recurrence relations for the computed residuals, which control the convergence of the method, are perturbed within safe bounds. Numerical examples are presented to demonstrate the effectiveness of this new residual replacement scheme.

Convergence study of the Chorin-Marsden formula

Longan Ying
School of Mathematics, Peking University

We study the convergence problem of the Chorin-Marsden formula for viscous splitting of the Navier-Stokes equations. Some results with respect to half plane and convex domains are given.

On the truncated conjugate gradient method

Ya-xiang Yuan
Institute of Computational Mathematics and Scientific/Engineering Computing, Chinese Academy of Sciences

In this paper, we consider the truncated conjugate gradient method for minimizing a convex quadratic function subject to a ball trust region constraint. It is shown that the reduction in the objective function by the solution obtained by the truncated CG method is at least half of the reduction by the global minimizer in the trust region.

One-way wave equations and their applications in seismic exploration problems and potential-inversion problems

Zhang Guanquan

Institute of Computional Mathematics and Scientific/Engineering Computing, Chinese Academy of Sciences

As well known the general solution of one-dimensional wave equation with constant propagation velocity consists of two traveling waves which propagate in one direction. The equations governing these waves are called one-way equations, sometimes right-going (or downngoing) and left-going (or upcoming) wave equations. In the variable velocity media the one-dimensional wave equation

$$\frac{1}{c^2(z)}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial z^2} = 0$$

can be transformed to the equivalent system of coupled equations for downgoing and upcoming waves

$$\begin{cases} (\frac{1}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial z})D(t,z) + \frac{c_z}{2c}(D+U) = 0, \\ (\frac{1}{c}\frac{\partial}{\partial t} - \frac{\partial}{\partial z})U(t,z) - \frac{c_z}{2c}(D+U) = 0, \end{cases}$$

where

$$\begin{aligned} D(t,z) &= \frac{1}{2}(\frac{1}{c}\frac{\partial}{\partial t} - \frac{\partial}{\partial z})p(t,z), \\ U(t,z) &= \frac{1}{2}(\frac{1}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial z})p(t,z), \end{aligned}$$

In multidimensional case by using some pseudo-differential operator Λ, called square-root operator, the similar system of coupled equations for downgoing and upcoming waves can be obtained. The square-root operator can be represented as an integro-differential (rational) operator. In such a way the obtained coupled system becomes a system of integro-differential equations. It can be proved that this system is equivalent to the original wave equation in the sense of high-frequency approximation. The numerical solution for thesystem of integro-differential equations can be implemented by suitably constructed difference methods. The numerical tests to solve the corresponding approximate system are performed for the seismic exploration problems to image the subsurface structure and for the inverse problems of wave equation to recovery the potential.

Mixed Elements for Linear Stokes Equations and Multigrid Solvers

Shangyou Zhang
Department of Mathematical Sciences, University of Delaware

A natural mixed finite element method for the linear Stokes equations in the velocity-pressure formulation is to approximate the velocity by continuous piecewise polynomials of degree (k+1) and to approximate the pressure by discontinuous piecewise polynomials of degree k.

This talk is on the stability of such mixed finite element methods, and on the constant rate of convergence for the multigrid method when applied to solve some the mixed element equations where the polynomial degree k is high.

Here we use an iterated penalty method to treat the divergence-free constraint so that the multigrid method is applied to a positive definite system.

We also discuss the recovery of the pressure function.

Incomplete Orthogonalization Preconditioner for Solving Large and Dense Linear Systems In Semidefinite Programming

Shao-Liang Zhang and Kazuhide Nakata
Department of Applied Physics, University of Tokyo
Masakazu Kojima
Dept. of Mathematical and Computing Sciences, Tokyo Institute of Technology

There is a strong need for the fast solution of large and dense linear systems which arise from Semidefinite Programming using the primal-dual interior-point method. Since direct methods are often too expensive in terms of computer memory and CPU-time requirements, then the only alternative is to use iterative methods. Here, an incomplete orthonalization preconditioner for the conjugate gradient method will be proposed. The efficient feature of the preconditioner will be confirmed by several unmerical experiments.

Superconvergent Finite Element Approximations of Problems with Boundary Layers

Zhimin Zhang
Dept of Mathematics, Texas Tech University

In scientific computing, we often encounter problems with boundary layers, solution varies rapidly within very thin layer regions near the boundary. The finite element method on quasi-uniform meshes fails to catch the rapid change of the solution in boundary layers, and this failure in turn "pollutes" the numerical approximation on the whole domain.

In this work, finite element methods on anisotropic meshes are studied. Model problems are singularly perturbed reaction-diffusion and convection-diffusion equations in the one and two-dimensional settings. It is shown that under properly designed meshes, which is highly refined near the boundary, the finite element method not only converges with near optimal rate in the L^2 norm, but also superconverges in a discrete energy norm. Numerical test results are provided to confirm the theory. In addition, the error in the L^∞ norm is also analyzed and much improved error bounds (comparing with other error bounds in the L^∞ norm in the literature) are given in this context.

Implicit, Nonparametric Shape Reconstruction from Unorganized Data

Hongkai Zhao
Department of Mathematics, Stanford University

In this talk we consider a fundamental visualization problem which arises in computer vision and numerical simulation. The problem is to find a curve in two dimensions, or a surface in three dimensions which can be regarded as the shape represented by a set of unorganized points, and/or curves, and/or surface pieces. We do not assume any knowledge of the ordering, connectivity or topology of the data sets or of the true shape. Only the location of each point or general Hausdorff distance to the data set is known. The key idea in our approach is to find an implicit nonparametric representation of the curve or surface on a fixed rectangular grid. With this representation of surfaces we can trivially find the closest point and distance from any point to the surface (useful in illumination and many other applications), intersection curve of two surfaces and any Boolean operation. We use a version of the variational level set method to interpret the desired shape as one or more elastic membranes attached to these points. We prove that there exists a local minimum which includes the data sets before the shape disappears. In 3D the final surface satisfies an interesting relationship involving its curvature and direction of its normal, i.e., it behaves like a weighted minimal surface. It looks much more pleasing than the usual polyhedron approach when the data points are not very dense or not evenly spaced. Since this is a variational approach it is also robust subject to some extent of noises. These two properties can be extremely useful for 3D reconstruction from photo images instead of from expensive but precise laser scanning data. The method allows for the addition of new data points and dynamic motion easily.

Application of SSM to Evaluating Two-factor Convertible Bonds

You-lan Zhu
Dept of Mathematics, University of North Carolina at Charlotte

From the mathematical point of view, to evaluating a convertible bond with continous dividend is to solve a free boundary problem of a parabolic equation. A two-factor convertible bond problem means that the problem is a two dimensional problem. In this work we combine the Singularity-Separating Method and the spectral method, so that highly accurate solution can be gotten quickly.

Numerical inversion of thermal source density and initial data

Jun Zou
Department of Mathematics, The Chinese University of Hong Kong

Given the temperature measurements on a subset of the domain occupied by some thermal material and at a fixed time $\theta > 0$, we will investigate some numerical inversion algorithms for the recovery of the thermal source density and of initial temperature of the material.