Solution for HW2, MATH3805

1. (a)

$$Y = X\beta + \epsilon$$

$$X = \begin{pmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{pmatrix}^T, Y = \begin{pmatrix} y_1 & y_2 & \dots & y_n \end{pmatrix}^T$$

Then we can write the derivatives of the sum of squared errors and set it equal to 0. Then the normal equation is

$$X^\top X\hat{\beta} - X^\top Y = 0$$

(b)

(c) $\begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix} = \left(X^T X\right)^{-1} X^T Y$

$$\left(X^T X\right)^{-1} X^T Y = \frac{1}{n\sum x_i^2 - \left(\sum x_i\right)^2} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{pmatrix} \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix}$$

So we have

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \bar{x}\hat{\beta}_1$$

where $\bar{y} = \sum_{i=1}^n y_i$ and $\bar{x} = \sum_{i=1}^n x_i$ That is

$$\hat{\beta}_0 = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$\hat{\beta}_1 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

(d)

(e)

$$\begin{bmatrix} \text{var}\left(\hat\beta_0\right) & \text{cov}\left(\hat\beta_0,\hat\beta_1\right) \\ \text{cov}\left(\hat\beta_0,\hat\beta_1\right) & \text{var}\left(\hat\beta_1\right) \end{bmatrix} = \sigma^2\left(X^TX\right)^{-1}$$

$$\text{var}\left(\hat\beta_0\right) = \frac{\sigma^2\sum x_i^2}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$\text{var}\left(\hat\beta_1\right) = \frac{n\sigma^2}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

2. Exercise B.16

   (a) $Y = (4,3,3,1,-1)^T, X = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{pmatrix}^T$

   (b) $X^TX = \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix}, X^TY = \begin{pmatrix} 10 \\ -12 \end{pmatrix}$

   (c) $\hat\beta = \left(X^TX\right)^{-1}X^TY = \begin{pmatrix} 2 \\ -1.2 \end{pmatrix}$

   (d) $\hat y = 2 - 1.2x$

3. Exercise B.19

$$H_0:\beta_1 = 0 \quad \text{vs} \quad H_1:\beta_1 \neq 0$$

$$s = 0.7303$$

$$t = \frac{\hat\beta_1}{s_{\hat\beta_1}} = \frac{\hat\beta_1}{s\sqrt{c_{11}}} = -5.1962$$

   p-value: 0.01385. or Reject region: $|t| > t_{\frac{\alpha}{2},3}$

   Reject $H_0, \beta_1 \neq 0$

4. Exercise B.22

$$E(y|x=1) = 0.8$$

$$\text{var}(E[y|x=1]) = s^2x_p^T\left(X^TX\right)^{-1}x_p = 0.16$$

   90% confidence interval for mean is $(-0.1413, 1.7413)$

   $E(y|x=1)$ will fall into the confidence interval with probability 90%.

   Exercise B.23

   $\text{var}(y|x=1) = s^2 + s^2x_p^T\left(X^TX\right)^{-1}x_p = 0.6933$

   90% prediction interval is $(-1.1593, 2.7593)$

   y will fall into the prediction interval with probability 90%.

5. Exercise B.30

(a)

$$y = \begin{pmatrix} 5.2 \\ 0.3 \\ -1.2 \\ 2.2 \\ 6.2 \\ 5 \\ -0.1 \\ -1.1 \\ 2.0 \\ 6.1 \end{pmatrix}, X = \begin{pmatrix} 1 & -2 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & -2 \\ 1 & 1 & -1 \\ 1 & 2 & 2 \\ 1 & -2 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & -2 \\ 1 & 1 & -1 \\ 1 & 2 & 2 \end{pmatrix}$$

(b) $\hat{\beta} = \left(X^T X\right)^{-1} X^T Y = (2.46, 0.41, 1.6143)^T$

$y = 2.46 + 0.41x_1 + 1.6143x_2$

(c) $SSE = \left(Y - X\hat{\beta}\right)^T \left(Y - X\hat{\beta}\right) = 2.4363$

$s^2 = SSE/(7) = 0.3480$

(d)

$$F\text{−}value = \frac{\left[(y-\bar{y})^T(y-\bar{y}) - SSE\right]/2}{SSE/7} = \frac{(78.7640 - 2.4363)/2}{2.4363/7} = 109.6534$$

$$p - \text{value} = 1 - F_{(2,7)}(109.6661) = 0.0000.\text{or}$$
$$F - \text{value} > F^{-1}_{(2,7)}(0.95) = 4.74$$

Reject H_0. The model contribute information for predicting y.

(e)

$$R^2 = \frac{SSR}{SSR + SSE} = 0.969$$

96.9% of y's variation in samples can be explained by the model.

(f)

$$\text{var}(\hat{\beta}) = s^2 \left(X^T X\right)^{-1} = \begin{pmatrix} 0.0348 & 0 & 0 \\ 0 & 0.0174 & 0 \\ 0 & 0 & 0.0124 \end{pmatrix}$$

$$t - \text{value} = \frac{\hat{\beta}}{s_{\hat{\beta}}} = \frac{0.41}{\sqrt{0.0174}} = 3.1082$$

p-value: 0.01713. or $|t\text{−value}| > 2.3646$

Reject $H_0, \beta_1 \neq 0$

The practical implication is the extrusion pressure will effect the strength of the new plastic.

(g)

$$\hat{y} = x\hat{\beta} = 4.8686$$
$$\text{Var}(E(\hat{y})) = s^2 x^T \left(X^T X\right)^{-1} x = 0.1541$$

90% confidence interval for mean is

$$\left(\hat{y} \pm t_{0.05,7}\sqrt{\text{Var}(E(\hat{y}))}\right) = (4.1248, 5.6124)$$

(h)

$$\text{Var}(\hat{y}) = s^2 + s^2 x^T \left(X^T X\right)^{-1} x = 0.5022$$

90% prediction interval is

$$\left(\hat{y} \pm t_{0.05,7}\sqrt{\text{Var}(\hat{y})}\right) = (3.5260, 6.2111)$$

6. Exercise 4.11

(a)

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4$$

(b)

$$\hat{y} = 21087.951 + 108.451x_1 + 557.910x_2 - 340.166x_3 + 85.681x_4$$

(c) Holding the value of the remaining variables fixed, the mean change in y for every 1-unit increase in x_1 is 108.451. For x_2, the mean change is 557.910. For x_3, the mean change is -340.166. For x_4, the mean change is 85.681.

(d) SPSS output:
T-statistic for β_1 : 1.222 and p-value: $0.236 > \alpha$. we can not reject H_0. So x_2 is a useless predictor.

(e) $R^2 = 0.912$ $R_a^2 = 0.894$. R^2 represents that 91.2% variation of y could be explained. And R_a^2 represents that 89.4% variation in y can be explained when considering the sample size and the number of parameters. Like R^2, adjusted R^2 also evaluates how many percent of the variation in y can be explained by the multiple regression model. However, unlike R^2, adjusted R^2 takes into account (adjusted for) both the sample size and the number of parameters such that a model of more parameter will have a heavy penalty so that adjusted R^2 cannot be forced to 1 by simply adding more and more parameters. Thus the R_a^2 will be preferred as it takes the sample size and the number of parameters into account.

(f) F-value = 51.720 p-value = 0.0000.
reject H_0, at least one should not be 0.

Model: MODEL1
Dependent Variable: RFEWIDTH

Number of Observations Read	25
Number of Observations Used	25

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	49163	12291	51.72	<.0001
Error	20	4752.76913	237.63846		
Corrected Total	24	53915			

Root MSE	15.41553	R-Square	0.9118
Dependent Mean	88.32000	Adj R-Sq	0.8942
Coeff Var	17.45417		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	21088	18553	1.14	0.2691
REDSHIFT	1	108.45084	88.73979	1.22	0.2359
LINEFLUX	1	557.90980	315.99021	1.77	0.0927
LUMINOSITY	1	-340.16553	320.76260	-1.06	0.3016
AB1450	1	85.68102	6.27334	13.66	<.0001

Figure 1: SAS output without intercept for Exercise 4.11

7. Exercise 4.13

(a) The first order model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5$$

where RPM is x_1, CPRATIO is x_2, INLETTEMP is x_3, EXHTEMP is x_4, AIRFLOW is x_5, HEATRATE is y.

(b)

$$\hat{y} = 13614 + 0.08879x_1 + 0.3519x_2 - 9.2009x_3 + 14.3939x_4 - 0.8480x_5$$

(c) β_0 represent the y-intercept of the line and β_1 represent the slope. Holding the value of the remaining variables fixed, the mean change in y for every 1-unit increase in RPM x is 0.08879. For CPRATIO (x_2), the mean change is 0.3519. For INLETTEMP (x_3), the mean change is 9.2009. For EXHTEMP (x_4), the mean change is 14.3939. For AIRFLOW (x_5), the mean change is -0.8480.

(d) $s = 458.8284$. It values the variation of y, is an estimator of σ. mean $\pm 2s$ provide a rough confidence interval.

(e) The adjusted R^2 is 0.9172. R_a^2 represents that 91.72% variation in y can be explained when considering the sample size and the number of parameters. Like R^2, adjusted R^2 also evaluates how many percent of the variation in y can be explained by the multiple regression model. However, unlike R^2, adjusted R^2 takes into account (adjusted for) both the sample size and the number of parameters such that a model of more parameter will have a heavy penalty so that adjusted R^2 cannot be forced to 1 by simply adding more and more parameters. Thus the R_a^2 will be preferred as it takes the sample size and the number of parameters into account.

(f) F-value is 147.30, and P -value < 0.0001. So the overall model is useful.

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: HEATRATE

Number of Observations Read	67
Number of Observations Used	67

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	155055273	31011055	147.30	<.0001
Error	61	12841935	210524		
Corrected Total	66	167897208			

Root MSE	458.82843	R-Square	0.9235
Dependent Mean	11066	Adj R-Sq	0.9172
Coeff Var	4.14613		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	13614	870.01294	15.65	<.0001
RPM	1	0.08879	0.01391	6.38	<.0001
CPRATIO	1	0.35190	29.55568	0.01	0.9905
INLETTEMP	1	-9.20087	1.49920	-6.14	<.0001
EXHTEMP	1	14.39385	3.46095	4.16	0.0001
AIRFLOW	1	-0.84796	0.44211	-1.92	0.0598

Figure 2: SAS output without intercept for Exercise 4.13

Exercise 4.24

(a) Under the condition RPM=7500, CPRATIO=13.5, INLETTEMP=1000, EXHTEMP =525, AIRFLOW=10.0, y will appear in the interval $(11599.6, 13665.5)$ with probability 95%.

(b) Under the condition RPM=7500, CPRATIO=13.5, INLETTEMP=1000, EXHTEMP =525, AIRFLOW=10.0, $E(y)$ will appear in the interval $(12157.9, 13107.1)$ with probability 95%.

(c) Yes. The confidence interval only considers variance of $X\beta$, but the prediction interval should consider the sum of two variance of $X\beta$ and residual.

Exercise 4.32

(a) The linear order model is:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 \beta_5 x_5 + \beta_6 x_2 x_5 + \beta_7 x_3 x_5$$

where RPM is x_1, INLETTEMP is x_2, EXHTEMP is x_3, CPRATIO is x_4, AIRFLOW is x_5.

(b)

$$\begin{aligned}\hat{y} =& 13646 - 0.04560x_1 - 12.6752x_2 + 23.0025x_3 - 3.0227x_4 + 1.2882x_5 \\ &+ 0.0162x_2x_5 - 0.0414x_3x_5\end{aligned}$$

(c)

$$H_0 : \beta_6 = 0 \quad vs \quad H_1 : \beta_6 \neq 0$$

The t-statistic for x_2x_5 is 4.40 and p-value is less than 0.0001. So inlet temperature and air flow rate interact is useful to explain heat rate.

(d)

$$H_0 : \beta_7 = 0. \quad vs \quad H_1 : \beta_7 \neq 0$$

The t-statistic for x_3x_5 is -3.77 and p-value is less than 0.0004. So exhaust temperature and air flow rate interact is useful to explain heat rate.

(e) part linear relationship between heat rate y and temperature (both inlet and exhaust) depends on air flow rate.

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: HEATRATE

Number of Observations Read	67
Number of Observations Used	67

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	7	158234406	22604915	138.02	<.0001
Error	59	9662802	163776		
Corrected Total	66	167897208			

Root MSE	404.69286	R-Square	0.9424
Dependent Mean	11066	Adj R-Sq	0.9356
Coeff Var	3.65694		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	13646	1068.17448	12.77	<.0001
RPM	1	0.04599	0.01602	2.87	0.0057
INLETTEMP	1	-12.67517	1.54155	-8.22	<.0001
EXHTEMP	1	23.00252	3.76778	6.11	<.0001
CPRATIO	1	-3.02265	26.41853	-0.11	0.9093
AIRFLOW	1	1.28815	3.56266	0.36	0.7190
IA	1	0.01615	0.00367	4.40	<.0001
EA	1	-0.04143	0.01098	-3.77	0.0004

Figure 3: SAS output without intercept for Exercise 4.32

Exercise 4.64

(a)
$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$$
where PRM is x_1, CPR is x_2

(b) H_0: $\beta_4 = \beta_5 = 0$ vs H_1: at least one are unequal to zero.

(c) Reduced:
$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$
Complete:
$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2$$

(d)
$$SSE_R = 25310639, SSE_C = 19370350, MSE_C = 317547$$

(e) $F = \frac{(SSE_R - SSE_C)/2}{\text{SSE}_C/61} = 9.3534$

(f) $F^{-1}(0.9|2, 61) = 2.3917$.
Thus the rejection region was $F > 2.3917$.

(g) the curvature terms in the complete second-order model are useful.

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: HEATRATE

Number of Observations Read	67
Number of Observations Used	67

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	142586570	47528857	118.30	<.0001
Error	63	25310639	401756		
Corrected Total	66	167897208			

Root MSE	633.84239	R-Square	0.8492
Dependent Mean	11066	Adj R-Sq	0.8421
Coeff Var	5.72761		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	12065	418.52997	28.83	<.0001
RPM	1	0.16969	0.03467	4.89	<.0001
CPRATIO	1	-146.06557	26.65913	-5.48	<.0001
RC	1	-0.00242	0.00312	-0.78	0.4401

Figure 4: SAS output without intercept for Exercise 4.64

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: HEATRATE

Number of Observations Read	67
Number of Observations Used	67

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	148526859	29705372	93.55	<.0001
Error	61	19370350	317547		
Corrected Total	66	167897208			

Root MSE	563.51284	R-Square	0.8846
Dependent Mean	11066	Adj R-Sq	0.8752
Coeff Var	5.09209		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	15583	1142.85985	13.63	<.0001
RPM	1	0.07823	0.11044	0.71	0.4814
CPRATIO	1	-523.13391	103.37571	-5.06	<.0001
RPMSQ	1	-1.80598E-7	0.00000197	-0.09	0.9272
CPRSQ	1	8.84007	2.16320	4.09	0.0001
RC	1	0.00445	0.00558	0.80	0.4282

Figure 5: SAS output without intercept for Exercise 4.64

8. Exercise 4.26

   (a) F-value= 226.35 and p-value< 0.001, the overall model is useful.
   (b) t-value is -3.09. Under $\alpha = 0.05$, this variable is significant.
   (c) Given $x_2 = 1$, $\hat{y} = 0.044 + 0.269x_1$
   (d) Given $x_2 = 7$, $\hat{y} = 0.308 - 0.673x_1$
   (e) In part c, y is positive related to x. when x increase, y will increase. For part d, y is negative related to x.

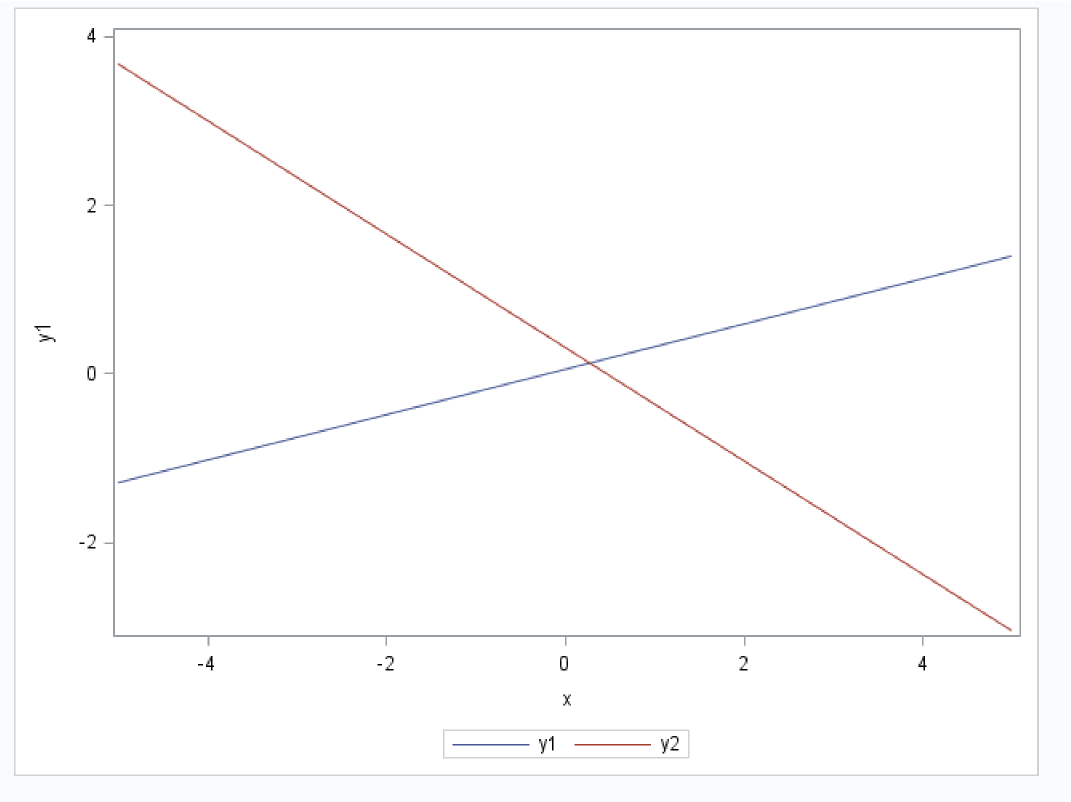

Figure 6: SAS output without intercept for Exercise 4.26

9. Exercise 4.37

   (a) not exact linear relationship.

   (b)

$$H_0 : \beta_2 = 0 .vs. H_1 : \beta_2 \neq 0$$

   t-value is 2.69 and p-value is 0.031.
   Under $\alpha = 0.10$, the quadratic variable is significant.

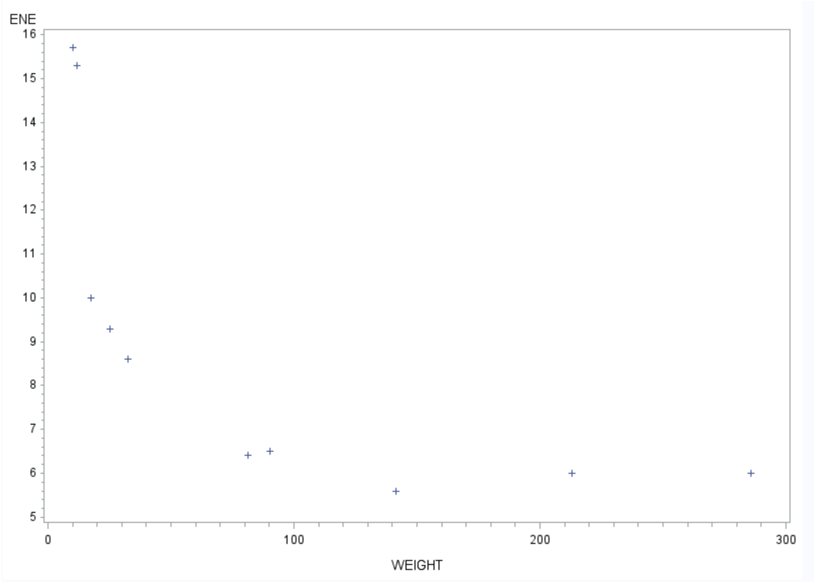

Figure 7: Scatter plot for Exercise 4.37

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: ENE

Number of Observations Read	10
Number of Observations Used	10

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	94.65852	47.32926	9.83	0.0093
Error	7	33.70548	4.81507		
Corrected Total	9	128.36400			

Root MSE	2.19433	R-Square	0.7374
Dependent Mean	8.94000	Adj R-Sq	0.6624
Coeff Var	24.54504		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	13.71274	1.30625	10.50	<.0001
WEIGHT	1	-0.10184	0.02881	-3.53	0.0095
WEIGHTSQ	1	0.00027348	0.00010160	2.69	0.0310

Figure 8: SAS output without intercept for Exercise 4.37

10. Exercise 4.59

    (a)

    $$\hat{y} = 80.22 + 156.5x_1 + 272.84x_2 + 760.1x_1x_2 - 42.3x_1^2 + 47x_1^2x_2$$

    (b)

    $$H_0 : \beta_1 = \ldots = \beta_5 = 0 \quad vs \quad H_1 : \exists i, \beta_i \neq 0$$

    F-value is 417.05, and p-value < 0.0001. So the overall model is useful.

    (c) There is no enough evidence to indicate that y is curvilinearly related to x_1. We should compare with a reduced model:

    $$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

    Exercise 4.69

    (a) In the null model, we consider the curvilinearly relationship in two ways: 1. x_1 is curvilinearly related to y directly; $2x_1$ is curvilinearly related to y based on x_2.

    (b) H_0: $\beta_4 = \beta_5 = 0$ vs H_1: at least one is not 0.

    (c) The curvilinearly variables of x_1 are useless.

    (d) $F = \frac{(SSE_R - SSE_C)/2}{SSE_C/(n-6)} = \frac{(89171-88819)/2}{88819/30} = 0.0594$

11. Exercise 4.88

    (a)

    $$\hat{y} = 10.6590 - 0.28161x_1 + 0.00267x_1^2$$

    (b) $R_a^2 = 0.8770$(1 point) the percentage of variation of sample with penalty of degree can be explained by the model.

    (c) $s = 4.5486$, estimate σ, reflect the variation of y.

    (d)

    $$H_0 : \beta_1 = \beta_2 = 0 .vs. H_1 : \exists i, \beta_i \neq 0$$

    F-value is 33.08 , and P -value is 0.0003. So the overall model is useful.

    (e)

    $$H_0 : \beta_2 = 0 .vs. H_1 : \beta_2 \neq 0$$

    t-value is 2.13 and P-value is $0.0706 > 0.05$. The evidence is not enough to conclude that the percentage improvement y increase more quickly for more costly fleet modifications than for less costly fleet modifications.

(f)

$$H_0 : \beta_3 = \ldots = \beta_5 = 0 . vs . H_1 : \exists 3 \leq i \leq 5, \beta_i \neq 0$$

$$F = \frac{\left(SSE_R - SSE_C\right)/3}{\mathrm{SSE}_C\,/(n-6)} = 0.3301, F^{-1}(0.95|3,4) = 6.5914$$

The type of base x_2 is useless.

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: PERCENT

Number of Observations Read	10
Number of Observations Used	10

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	1368.77501	684.38750	33.08	0.0003
Error	7	144.82499	20.68928		
Corrected Total	9	1513.60000			

Root MSE	4.54855	R-Square	0.9043
Dependent Mean	17.20000	Adj R-Sq	0.8770
Coeff Var	26.44504		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	10.65904	14.55009	0.73	0.4876
COST	1	-0.28161	0.28088	-1.00	0.3494
COSTSQ	1	0.00267	0.00125	2.13	0.0706

Figure 9: SAS output without intercept for Exercise 4.88

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: PERCENT

Number of Observations Read	10
Number of Observations Used	10

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	1397.51481	279.50296	9.63	0.0238
Error	4	116.08519	29.02130		
Corrected Total	9	1513.60000			

Root MSE	5.38714	R-Square	0.9233
Dependent Mean	17.20000	Adj R-Sq	0.8274
Coeff Var	31.32059		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	2.03190	22.64502	0.09	0.9328
COST	1	-0.10364	0.48513	-0.21	0.8413
BASE	1	49.76686	52.36930	0.95	0.3958
COSTSQ	1	0.00189	0.00234	0.81	0.4643
CB	1	-0.87476	0.93575	-0.93	0.4028
COSTSQBASE	1	0.00353	0.00398	0.89	0.4246

Figure 10: SAS output without intercept for Exercise 4.88