MATH 3826 Assignment 4

1. For the M/M/1 queue, compute

   (a) the expected number of arrivals during a service period and

   (b) the probability that no customers arrive during a service period.

   Hint: "Condition."

2. The manager of a market can hire either Mary or Alice. Mary, who gives service at an exponential rate of 20 customers per hour, can be hired at a rate of \$3 per hour. Alice, who gives service at an exponential rate of 30 customers per hour, can be hired at a rate of \$$C$ per hour. The manager estimates that, on the average, each customer's time is worth \$1 per hour and should be accounted for in the model. Assume customers arrive at a Poisson rate of 10 per hour

   (a) What is the average cost per hour if Mary is hired? If Alice is hired?

   (b) Find C if the average cost per hour is the same for Mary and Alice.

3. Suppose that a customer of the M/M/1 system spends the amount of time $x > 0$ waiting in queue before entering service.

   (a) Show that, conditional on the preceding, the number of other customers that were in the system when the customer arrived is distributed as $1 + P$, where P is a Poisson random variable with mean λ.

   (b) Let W_Q^* denote the amount of time that an M/M/1 customer spends in queue. As a by-product of your analysis in part (a), show that

$$P\{W_Q^* \le x\} = \begin{cases} 1 - \frac{\lambda}{\mu}, & \text{if } x = 0 \\ 1 - \frac{\lambda}{\mu} + \frac{\lambda}{\mu}(1 - e^{-(\mu-\lambda)x}), & \text{if } x > 0 \end{cases}$$

4. A facility produces items according to a Poisson process with rate λ. However, it has shelf space for only k items and so it shuts down production whenever k

items are present. Customers arrive at the facility according to a Poisson process with rate μ. Each customer wants one item and will immediately depart either with the item or empty handed if there is no item available.

(a) Find the proportion of customers that go away empty handed.

(b) Find the average time that an item is on the shelf.

(c) Find the average number of items on the shelf.

5. Customers arrive to a two server system in accordance with a Poisson process with rate λ. Server 1 is the preferred server, and an arrival finding server 1 free enters service with 1; an arrival finding 1 busy but 2 free, enters service with 2. Arrivals finding both servers busy do not enter. A customer who is with server 2 at a moment when server 1 becomes free, immediately leaves server 2 and moves over to server 1. After completing a service (with either server) the customer departs. The service times at server i are exponential with rate $\mu_i, i = 1, 2$.

   (a) Define states and give the transition diagram.

   (b) Find the long run proportion of time the system is in each state.

   (c) Find the proportion of all arrivals that enter the system.

   (d) Find the average time that an entering customer spends in the system.

   (e) Find the proportion of entering customers that complete service with server 2.

6. Customers arrive at a two-server system according to a Poisson process having rate $\lambda = 5$. An arrival finding server 1 free will begin service with that server. An arrival finding server 1 busy and server 2 free will enter service with server 2. An arrival finding both servers busy goes away. Once a customer is served by either server, he departs the system. The service times at server i are exponential with rates μ_i , where $\mu_1 = 4, \mu_2 = 2$.

   (a) What is the average time an entering customer spends in the system?

   (b) What proportion of time is server 2 busy?