3. Appendix

Page 12, **Example.**
By using the MDP approach, we can compute the following:

$$v_i(n) = \max_{k \in A_i} \{ q_i^{(k)} + \alpha \sum_j P_{ji} v_j(n-1) \}$$

and have

$$v_1(4) = \max \left\{ \begin{array}{c} 8 + 0.9 \times [0.4 \times v_1(3) + 0.6 \times v_2(3)] \\ 7 + 0.9 \times [0.8 \times v_1(3) + 0.2 \times v_2(3)] \\ 5 + 0.9 \times [1 + v_1(3)] \end{array} \right\};$$

$$v_2(4) = \max \left\{ \begin{array}{c} 4 + 0.9 \times [0.1 \times v_1(3) + 0.9 \times v_2(3)] \\ 3 + 0.9 \times [0.4 \times v_1(3) + 0.6 \times v_2(3)] \\ 1 + 0.9 \times [0.8 \times v_1(3) + 0.2 \times v_1(3)] \end{array} \right\};$$

$$v_1(3) = \max \left\{ \begin{array}{c} 8 + 0.9 \times [0.4 \times v_1(2) + 0.6 \times v_2(2)] \\ 7 + 0.9 \times [0.8 \times v_1(2) + 0.2 \times v_2(2)] \\ 5 + 0.9 \times [1 \times v_1(2)] \end{array} \right\};$$

$$v_2(3) = \max \left\{ \begin{array}{c} 4 + 0.9 \times [0.1 \times v_1(2) + 0.9 \times v_2(2)] \\ 3 + 0.9 \times [0.4 \times v_1(2) + 0.6 \times v_2(2)] \\ 1 + 0.9 \times [0.8 \times v_1(2) + 0.2 \times v_1(2)] \end{array} \right\};$$

Page 12, **Example.** continuous

$$v_1(2) = \max \left\{ \begin{array}{c} 8 + 0.9 \times [0.4 \times v_1(1) + 0.6 \times v_2(1)] \\ 7 + 0.9 \times [0.8 \times v_1(1) + 0.2 \times v_2(1)] \\ 5 + 0.9 \times [1 \times v_1(1)] \end{array} \right\};$$

$$v_2(2) = \max \left\{ \begin{array}{c} 4 + 0.9 \times [0.1 \times v_1(1) + 0.9 \times v_2(1)] \\ 3 + 0.9 \times [0.4 \times v_1(1) + 0.6 \times v_2(1)] \\ 1 + 0.9 \times [0.8 \times v_1(1) + 0.2 \times v_1(1)] \end{array} \right\};$$

$$\left\{ \begin{array}{l} v_1(1) = \max\{8, 7, 5\} = 8, p_1(1) = 1; \\ v_2(1) = \max 4, 3, 1 = 4, p_2(1) = 1. \end{array} \right.$$

With the results from the last equations, we can solve for other values by backward substitution. Let $p_i(n) = k^*$ such that

$$\max_{k \in A_i} \left\{ q_i^{(k)} + \alpha \sum_j p_{ji}^{(k)} v_j(n-1) \right\} = q_i^{(k^*)} + \alpha \sum_j p_{ji}^{(k^*)} v_j(n-1)$$

Page 12, **Example.** continuous

then $p_i(n)$ actually keeps track of the optimal policy for every single period. We can summarize all results from the calculations in the following Tables

Table 3.1: A summary of results

n	1	2	3	4
$v_1(n)$	8	13.48	18.15	22.27
$v_2(n)$	4	8.04	12.19	16.27
$p_1(n)$	1	2	2	2
$p_2(n)$	1	2	2	3

Table 3.2: A summary of results

n	1	2	3	4
$v_1(n)$	8	11.36	13.25	14.35
$v_2(n)$	4	6.64	8.27	9.26
$p_1(n)$	1	1	2	2
$p_2(n)$	1	1	1	1

Page 12, **Example.** continuous

Since the on-line gaming company started from having a low volume of players (State 2), the optimal policy for the company is as follows: with 4 more years left, choose Alternative 3 (fully upgrade); then use Alternative 2 (regular maintenance) for two consecutive years; and finally, use Alternative 1 (no action) when there is only 1 year left.

Note that the optimal policy may vary depending on the value of the discount factor. For instance, if in this example, we have a discount factor of 0.6, then we have different results as summarized in Table 3.2. If the company starts with a low volume of players, the optimal policy is to stay with Alternative 1 (no action). We leave it as an exercise for the reader to device the results themselves.