Summary and Review

Lecture Note 1

- Sample Space S: the set of all possible outcome of an experiment.
- Event E: any subsetof the sample space S is known as an event.
- Complement E^c: consist of all outcomes in the sample space S that are not in E.
- Probabilities $P(E)$ defined on Events satisties the three conditions. (Page 6, Lecture Note 1)
- Operations: $P(E \cup F), P(E \cap F)$.

- Conditional Probability $P(E|F) = \frac{P(E \cap F)}{P(F)}$

- Two events E and F are said to be independence if $P(E \cap F) = P(E)P(F)$ or $P(E|F) = P(E)$.

- Bayes' Formula: $P(E) = P(E|F)P(F) + P(E|F^c)(1 - P(F))$ and

$$P(F_j|E) = \frac{P(E \cap E_j)}{P(E)} = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)F(F_i)}$$

- Random variables: These quantities of interest, or more formally, these real-valued functions defined on the sample space.

- Discrete random variables, Continuous random variables.

- Cumulative distribution function. (Page 26, Lecture Note 1)

- The Bernolli Random Variable, The Binomial Random Variable (Page 28, Lecture Note 1)
- The Geometric Random Variable, The Poisson Random Variable (Page 29, Lecture Note 1)
- Density function for Continuous Random variable (Page 30, Lecture Note 1)
- The uniform random variable, The Exponential Random Variables (Page 31, Lecture Note 1)
- The Gamma Random Variables, Normal Random Variables (Page 32, Lecture note 1)

- Expectation of a random variable for discrete random variable and continuous random variable. (Page 33, 34, Lecture note 1)

- Jointly Distribution of random variables, marginal distribution of random variables

- Independent random variables $P(X \le a, Y \le b) = P(X \le a)P(Y \le b)$ and $f(x,y) = f(x)f(y)$, and for any functions h and g, $\mathrm{E}[g(X)h(Y)] = \mathrm{E}[g(X)]\mathrm{E}[h(Y)]$.

- Covariance and Variance of random variables $\mathsf{Cov}(X,Y) = \mathrm{E}(X - \mathrm{E}X)(Y - \mathrm{E}Y) = \mathrm{E}(XY) - \mathrm{E}X\mathrm{E}Y$.

- Joint Probability Distribution of Functions of Random, Jacobian determinant Variables (page 43, 44, Lecture note 1)

- Moment Generating Functions (page 45, Lecture Note 1)

- Strong Law of large Number
- Central limit theory
- Condtional probability (***) (Lecture note 1, section 1.12)
- Condtional Expectation (***) (Lecture note 1, section 1.12)
- Computing Expectations and Probabilities by Conditioning (***) (Lecture note 1, section 1.12)

Lecture Note 2

- Definition of Stochastic process
- Discrete-time process and continuous-tiem process
- State space of a stochastic process
- Stationary process, strong stationary, weak stationary.
- autocovariance

- Definition of Markov Chain. Definition of State. (page 5, Lecture note 2)
- Transition probability and transition probability matrix of the markov chain (Page 5,6, Lecture not 2)
- Definition of Absobing states (Page 7, Example 3, Lecture Note 2)
- Chapman-Kolmogorov Equation (page 8)

- Classifiction of States
- Definition of Accessible and Communicate (page 11, Lecture Note 2)
- Defintion of Class for the state. (page 12, Lecture Note 2)
- Definition of irreducible (page 12, Lecture Note 2)
- Definition of Recurrent and Transient for the state (page 13, Lecture Note 2, Proposition 1 and Corollary 1.)

- Definition of positive recurrent and null recurrent (Page 17, Lecture Note 2)
- Definition of the long-run proportion of time that the Markov chain is in state (Page 18, 19 . Lecture note 2, Proposition 3, Theorem 1)
- Definition of Stationary probabilities of makove chain
- Definition of Limiting probability (Page 24,25, Lecture Note 2)
- Defintion of periodic, aperiodic, ergoic (Page 25, 26, Lecture Note 2)

- Mean Time Spend in Transient States, definition and calculation formulas (***) (Page 29, 30, 31, Lecture Note 2)
- Definition of Branching Process, calculation of Expectatin and variance of states for the branching process (Page 33-38, Lecture Note 2)
- Definition of Continuous-Time Markov Chains (Page 39, Lecture Note 2)
- Defintion of stationary or homogeneous transition probabilities (Page 39, Lecture Note 2)

- Definiton of Memoryless (Page 40, Lecture Note 2)
- The exponetnail distribution, mean, moment generating function, variance, memoryless. (Page 41-43, Lecture Note 2)
- Defintion of Counting process, indepdnent increments, statinary increments (Page 44, Lecture Note 2)
- Definition of Possion process (Page 45, 46Lecture Note 2)
- Defintion of the rate of Possion process (Page 45, Lecture Note 2)
- Mean Possion process, Properties of Posssion process , independent and stationary increments (Page 45, Lecture Note 2)

- Definition of Birth and Death process and its transition probability formula (pag 47, 48 Lecture Note 2)

- Definition of the Queueing System M/M/1, and A Multiserver Exponential Queueing System (Page 50,51, Lecture Note 2)

Lecture Note 3

- State and Action (Page 1, Lecture Note 3)
- Defintion of Policy (Page 2, Lecture Note 3)
- Definition of the resultant Markvo chain given the policy (Page 3, Lecture Note 3)
- Definition of the limiting (or steady-state) probability. (Page 3-4, Lecture Note 3)
- Defintion of Reward, expected average reward under a special policy.
- Maximizing Reward and Optimal Policy

- Definition of Markov Decision Processes for Customer Lifetime Value (page 8-9, Lecture Note 3)
- Definition of Discount factor (page 9, Lecture Note 3)
- Definition of Stationary Policy (page 11, Lecture Note 3)

Lecture Note 4

- Definitionof Queueing System (Page 1, Lecture Note 4)
- Defintion of fundamental quantities for queueing models: L, L_Q, W and W_Q (Page 2, Lecture Note 4)
- Defintion of average arrival rate of entering customers λ_a (Page 3, Lecture Note 4)
- Basic cost Identity Principle (***) (Page 3, Lecture Note 4)
- Generalized relationship or Little's formula $L = \lambda_a W$ and $L_Q = \lambda_a W_Q$ (Page 5, Lecture Note 4)
- Definition of the average amount of time a customer spends in service and average number of customers in service $= \lambda_a \mathrm{E}[S]$. (Page 5, Lecture Note 4)

- Definition of the limiting or long- run probability that there will be exactly n customers in the system P_n. (Page 6, Lecture Note 4)

- Definition of Steady-State Probability (Page 6, Lecture Note 4)

- Defintion of a_n, proportion of customers that find n in the system when they arrive, and d_n, proportion of customers leaving behind n in the system when they depart. (Page 6, Lecture Note 4)

- Relationship between P_n and a_n, d_n. (Page 7-8, Lecture Note 4)

- PASTA Principle: The result that Poisson arrivals see time averages (***) (Page 8, Lecture Note 4)

- Definition of $M/M/1$ queueing system: A Single-Server Exponential Queueing System. (Page 11, Lecture Note 4)

- The general principle: the rate at which the process enters state n equals the rate at which it leaves state n. (***) (Page 11, Lecture Note 4)

- Calculation of P_n, L, W, L_Q, W_Q for M/M/1 (Page 13-17, Lecture Note 4)

- Definition of A Single-Server Exponential Queueing System Having Finite Capacity (***) (Page 20, Lecture Note 4)

- Calculation of P_n, L, W, L_Q, W_Q for M/M/1 with finite capacity (***) (Page 20, Lecture Note 4)

- Definition of the M /M /1 Queueing System with Balking. (Pag 25, Lecture Note 4)

- Definition of The M /M /k Queueing System and Calculation of P_n (***) (Page 25-28, Lecture Note 4)

- Remarks (page 30-32, Lecture Note 4)

- Definition of Network of Queues (Page 33, Lecture Note 4)

- The balance equation for Network of Queues (Page 33-34, Lecture Note 4)

- Definition of M/G/1

- Cost rule: Each customer pays at a rate of y/unit time when his remaining service time is y, and calculation of L, W, L_Q, W_Q and Pollaczek-Khintchine formula (Page 36-40, Lecture Note 4)

- Definition of Busy and Idle Periods (Page 41, Lecture Note 4)

- Variation of the M/G/1. (Page 43-44, Lecture Note 4)

- Definition of the model G/M/1. (page 45, Lecture note 4)

- Definition of state space and markov chain for the G/M/1. (page 45, Lecture 4)

- Basic idea of calculating the transition probability P_{ij}, W, L, W_Q, L_Q for the G/M/1 system. (Page 46-49, Lecture Note 4)

- Basic idea of calculating Busy and Idle periods for the G/M/1.

- Definition of Multiserver Queues and Erlang's Loss System (Page 52, Lecture Note 4)

- Balance equations for Multiserver Queues system.

- The M/M/k queue (***)

- G/M/k queue, basic idea of deriving the transition probability.

- M/G/k queue, basic idea of calculating W_Q