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Jie Shen Tao Tang

Spectral and High-Order Methods with Applications

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Preface

This book expands lecture notes by the authors for a course on *Introduction of Spectral Methods* taught in the past few years at Penn State University, Simon Fraser University, the Chinese University of Hong Kong, Hong Kong Baptist University, Purdue University and the Chinese Academy of Sciences. Our lecture notes were also used by Prof. Zhenhuan Teng in his graduate course at Peking University.

The overall emphasis of the present book is to present some basic spectral and high-order algorithms together with applications to some linear and nonlinear problems that one frequently encounters in practice. The algorithms in the book are presented in a *pseudocode* format or with MATLAB or FORTRAN codes that contain additional details beyond the mathematical formulas. The reader can easily write computer routines based on the pseudocodes in any standard computer language. We believe that the readers learn and understand numerical methods best by seeing how algorithms are developed from the mathematical theory and then writing and testing computer implementations of them. For those interested in the numerical analysis of the spectral methods, we have also provided self-contained error analysis for some basic spectral-Galerkin algorithms presented in the book. Our aim is to provide a sufficient background on the implementation and analysis of spectral and high-order methods so that the readers can approach the current research literature with the necessary tools and understanding.

We hope that this book will be useful for people studying spectral methods on their own. It may also serve as a textbook for advanced undergraduate/beginning graduate students. The only prerequisite for the present book is a standard course in Numerical Analysis.

This project has been supported by NSERC Canada, National Science Foundation, Research Grant Council of Hong Kong, and International Research Team of Complex System of the Chinese Academy of Sciences. In writing this book, we have received much help from our friends and students. In particular, we would like to thank Dr. Lilian Wang of Nanyang Technical University of Singapore for his many contributions throughout the book. We are grateful to the help provided by Zhongzhi Bai of the Chinese Academy of Sciences, Weizhu Bao of National University of Singapore, Raymond Chan of Chinese University of Hong Kong, Wai Son Don of Brown

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A website relevant to this book can be found in

`http://www.math.hkbu.edu.hk/~ttang/PGteaching` or

`http://lsec.cc.ac.cn/~ttang/PGteaching`

We welcome comments and corrections to the book. We can be reached by email to

`shen@math.purdue.edu` (Shen) and `ttang@math.hkbu.edu.hk` (Tang).

Jie Shen
Purdue University

Tao Tang
Hong Kong Baptist University

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