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Theoretical and Numerical Aspects of Nonlinear Conservation Laws

Special session organizers:

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Titles & Abstracts

Long Time Behaviors of Solutions of Relaxation Hyperbolic Syatems with Non-Equilibrium Far-Field States

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Abstract

We study long-time behaviors of solutions of the relaxation hyperbolic system

$$
\begin{aligned}
u_t + f(u,v)_x &= 0 \\
v_t + g(u,v)_x &= h(u,v)
\end{aligned}
$$

with nonequilibrium far field states. This model was proposed by Liu, which consists of one conservation law and one rate equation. It is simple, yet general enough to study nonequilibrium phenomina. The flood flow with friction is a special case of this model.

The system is assumed to be hyperbolic with two characteristic speeds $\lambda_1 < \lambda_2$. We assume that $h_v < 0$ and the the equilibrium states are $v = v_*(u)$ where $h(u, v_*(u)) = 0$. The relaxed model is

$$u_t + f(u, v_*(u))_x = u_t + \lambda_*(u)u_x = 0.$$

It has been shown that the fluctuation causes an diffusion effect with diffusion coefficient $\nu = h_v^{-1}(\lambda_1 - \lambda_*)(\lambda_2 - \lambda_*)$ being assumed to be > 0.

In this talk, we assume the far field states are $(0, v_-)$ and $(0, v_+)$, but v_- and v_+ are not in equilibrium. We shall show that there will be mass injected from far field. This excess mass together with the initial mass form a diffusion wave time asymptotically. The amount of this excess mass is computed. The time-asymptotical diffusion wave has an explicit expression. The convergent rate in L^p-norm to this diffusion wave is obtained.

This is a joint work with Ming Mei of Vienna University of Technology, Australia

ON CONTACT OVERHEATING
and Other Computational Difficulties of Shock-Capturing Methods

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Abstract

Sudden compression of a gas flow due to shock reflection from a solid wall or sudden expansion due to an abrupt withdrawal of a piston from the gas are often associated with a phenomenon known generically as contact overheating (also called wall overheating). Conventional shock-capturing numerical methods accurately predict pressure and velocity, but over-predict temperature at the wall (a contact), hence the name. The problem has been studied since the dawn of simulating shock waves and has been revisited ever since.

We propose a new shock-capturing method that completely eliminates the contact overheating, while correctly predicting the whole flow field. The method uses: (a) the Lagranginan coordinates, (b) the shock-adaptive Godunov scheme and, (c) the entropy conservation equation in the smooth flow region. In addition, the new method also overcomes other computational difficulties commonly encountered by conventional shock-capturing methods, namely, the start-up errors, slow-moving shocks, low density flow, and the sonic point glitch.

An Explicit Divergence-free Upwindig Method for the MHD-equations

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Abstract

1 Introduction

Many flows, particularly astrophysical flows, are electrically conducting, and the electromagnetic forces can be of the same order as the hydrodynamic forces. The governing equations of **m**agneto**h**ydro**d**ynamics (MHD) basically merges the Euler equations of gas dynamics with the Maxwell equations of electromagnetics. A feature of the MHD-equation, in contrast to the Euler system, is the additional constrain $\operatorname{div}(\mathbf{B}) = 0$ on the magnetic field, which is expressed in the MHD-equation as $\partial_t (\operatorname{div}(\mathbf{B})) = 0$.

Based on the Method of Transport, we will present an explicit multi-dimensional upwind scheme for the MHD-equations, which conserves $\operatorname{div}(\mathbf{B})$ according to a numerical divergence-operator and whos timestep is fully local.

2 The Method of Transport

The **M**ethod **o**f **T**ransport (MoT) is a multi-dimensional upwinding scheme for non-linear systems of conservation laws. The method consists of two independent parts: The decomposition of the non-linear system of conservation laws into a set of linear systems an the proper solution of these linear systems.

A general system of conservation laws can be written as

$$\mathbf{U}_t + \nabla \cdot \underline{\mathbf{F}}(\mathbf{U}) = 0 \tag{1}$$

with the vector $\mathbf{U}$ of the conserved quantities and the flux matrix $\underline{\mathbf{F}}$. Decomposition means, that we look for k vectors $\mathbf{S}_i = \mathbf{S}_i(\mathbf{U})$ and velocities $\mathbf{a}_i = \mathbf{a}_i(\mathbf{U})$ that the two conditions

$$\mathbf{U} = \sum_{i=1}^{k} \mathbf{S}_i \qquad \text{and} \qquad \underline{\mathbf{F}}(\mathbf{U}) = \sum_{i=1}^{k} \mathbf{S}_i \mathbf{a}_i^T \tag{2}$$

holds. The actual linearisation consists in the decoupling of the functions $\mathbf{S}_i$ and $\mathbf{a}_i$ from $\mathbf{U}$. The system (1) can approximately solved by the set of linear equations

$$\begin{aligned} \left(\mathbf{S}_i(\mathbf{x},\tau)\right)_\tau + \nabla \cdot \left(\mathbf{S}_i(\mathbf{x},\tau)\, \mathbf{a}_i^T(\mathbf{x})\right) = 0 \qquad & i = 1, \dots, k \\ \mathbf{S}_i(\mathbf{x},0) = \mathbf{S}_i(\mathbf{U}(\mathbf{x},t)) \qquad & \mathbf{a}_i(\mathbf{x}) = \mathbf{a}_i(\mathbf{U}(\mathbf{x},t)). \end{aligned} \tag{3}$$

The solution after time Δt is

$$\mathbf{U}(\mathbf{x}, t + \Delta t) = \sum_{i=1}^{k} \mathbf{S}_i(\mathbf{x}, \Delta t).$$

We call (3) the advection form of (1). As a numerical scheme for the non-linear system we basically need a scheme for the scalar linear system.

3 The MHD-Equations

The MHD-equations can be written as conservation laws (1) with

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho\mathbf{u} \\ \mathbf{B} \\ E \end{pmatrix} \qquad \text{and} \qquad \underline{\mathbf{F}} = \begin{pmatrix} \rho\mathbf{u}^T \\ \rho\mathbf{u}\mathbf{u}^T + \left(p + \frac{|\mathbf{B}|^2}{2}\right)\underline{\mathbf{I}} - \mathbf{B}\mathbf{B}^T \\ \mathbf{B}\mathbf{u}^T - \mathbf{u}\mathbf{B}^T \\ \left(E + p + \frac{|\mathbf{B}|^2}{2}\right)\mathbf{u}^T - (\mathbf{u}\cdot\mathbf{B})\,\mathbf{B}^T \end{pmatrix} \tag{4}$$

where ρ denotes the density, $\mathbf{u}$ the velocity vector, $\mathbf{B}$ the magnetic field and E the total energy. The pressure p is related to the conserved quantities by the equation of state

$$E = \frac{p}{\gamma - 1} + \rho\frac{|\mathbf{u}|^2}{2} + \frac{|\mathbf{B}|^2}{2} \tag{5}$$

Even though a decompositions of the MHD-equations into advection form (3) can be found, it is not possible to construct a divergence-free scheme with this ansatz due to the following reasons. First, the advection form (3), like any other planar wave decomosition is unable to control the divergencen in the linear equations. Thus, the constrain can only be fulfilled after the avarraging process which is most likely to fail. Second, to construct an antisymmetric flux like the $\mathbf{B}$-component in (1) with planar waves, the leading order terms are allways symmetric and have to cancel. The smaller antisymmetric part is generated as a difference of large parts which leads to a huge numerical viscousity.

To take care of the special form of the equation for the magnetic field $\mathbf{B}$ we add a new advection type wave to the decomposotion that is able to control the divergence. We use l vectors $\mathbf{T}_j = \mathbf{T}_j\left(\mathbf{U}\right)$ and velocities $\mathbf{b}_j = \mathbf{b}_j\left(\mathbf{U}\right)$ such that

$$\mathbf{B} = \sum_{j=1}^{l}\mathbf{T}_j \qquad \text{and} \qquad \mathbf{B}\mathbf{u}^T - \mathbf{u}\mathbf{B}^T = \sum_{j=1}^{l}\left(\mathbf{T}_j\mathbf{b}_j^T - \mathbf{b}_j\mathbf{T}_j^T\right).$$

Again we decouple $\mathbf{T}_j$ from $\mathbf{B}$ and $\mathbf{b}_j$ from $\mathbf{U}$ and can approximately solve the original equation by the set of equations

$$\begin{gathered} \left(\mathbf{T}_j\left(\mathbf{x},\tau\right)\right)_\tau + \nabla\cdot\Big(\mathbf{T}_j\left(\mathbf{x},\tau\right)\mathbf{b}_j^T\left(\mathbf{x}\right) - \mathbf{b}_j\left(\mathbf{x}\right)\mathbf{T}_j^T\left(\mathbf{x},\tau\right)\Big) = 0 \qquad j = 1,\dots,l \\ \mathbf{T}_j\left(\mathbf{x},0\right) = \mathbf{T}_j\left(\mathbf{B}\left(\mathbf{x},t\right)\right) \qquad \mathbf{b}_j\left(\mathbf{x}\right) = \mathbf{b}_j\left(\mathbf{U}\left(\mathbf{x},t\right)\right) \end{gathered} \tag{6}$$

and the solution after time Δt is

$$\mathbf{B}\left(\mathbf{x}, t + \Delta t\right) = \sum_{j=1}^{l}\mathbf{T}_j\left(\mathbf{x},\Delta t\right).$$

In this approach, we need a solver for the advection equation (3) and additionally a solver for equations of the form (6), which preserves $\mathrm{div}\left(\mathbf{T}_j\right)$. Note, that this approch solves both problem in the previous section. First, each linear equation of typ (6) can control the divergence of the field. Second, we directly generate the antisymmetric tensor flux as leading order term. No symmetric parts have to cancel.

In the talk we will present the derivation of the numerical flux to solve equation (6) such that the discrete upwinding update meets the constrain ∂_t div $(\mathbf{T}_j) = 0$ exactly for a properly choosen discrete divergence operator. On some 1-D and 2-D examples the influence of different wave models or decompositions is illustrated.

This work joint with Ruedi Limacher and Michael Fey of ETH Eurich.

Multi-phase Flow with Reaction

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Abstract

In this paper, the effects of spilled oil on coastal ecosystem are discussed from the viewpoint of fluid dynamical studies. A few years ago, Russian tanker was stranded in the Japan Sea and oil flowed out into sea from the tanker. Spilled oil drifted on the shore, adhered to sand beach and penetrated into sand accompanied with the ebbing tide. Many volunteers and people from the Self-defense Force cooperated to remove spilled oil from seashore. At that time, some influences of spilled oil to exert on ecosystem were discussed from a lot of points of view. Such an incident mentioned above promoted strongly to organize national projects to study environmental pollution.

A part of this study is under cooperation with "Research for the Future Program" of the Japan Society for the Promotion of Science (Research theme: Evaluation and restoration of the effects of oil pollution on coastal ecosystem, Principal investigator: Prof. Mitsumasa Okada, Department of Environmental Science, Hiroshima University). Main themes included in this project are;

1. Process of drifting ashore of spilled oil.
2. Penetrating process of spilled oil into tidal flats and seabeds.
   - 2.1 Effect of wave motion to the penetration.
   - 2.2 Effect of tidal motion to the penetration.
   - 2.3 Transport process of oil in sand.
3. Decomposition process of spilled oil after the penetration.
4. Influence for the penetration to exert on soluble components in seawater.
5. Evaluation of influence for the penetration to exert on ecosystem and benthos.
6. Restorative techniques for the influence due to oil pollution.

This study focuses on theme 3 from fluid dynamical point of view. The phenomena of theme 3 describe the decomposition of penetrated oil into discharged materials by bacteria in the tidal flats, main components of which are water and soluble ones. These phenomena are mathematically modeled into a multi-phase flow formulation with reactions.

One of characteristics of this model is an appearance of velocity jump on the reaction surface between phases of oil and water based on the difference of densities among them. In order to treat this difficulty, Discontinuous Interface Generating Method (DIGM) was proposed by ourselves, an original idea of which has been invented through try and error of numerical experiments. Flow equations for all phases are unified into a single flow equation to construct a numerical model. The fictitious domain method plays an important role in this procedure. Also it should be noted that adhesion and penetration as characteristic properties of oil are taken into consideration through boundary conditions of friction type prescribed between oil and sand. Numerical results is presented.

New Nonstaggered, Nonoscillatory, High Resolution Central Schemes for Conservation Laws.

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Abstract

Central schemes for conservation laws and its related equations have been studied by Tadmor and his coworkers since 1980's. Compared to upwind schemes, the main advantage of central schemes is simplicity, because of no need of approximate Riemann solvers. However, the original version of central schemes is staggered in the sense that the grid alternates every other time step. Staggered schemes may increase the difficulty on dealing numerical boundary conditions. In this talk, I will address on two new nonstaggered, nonoscillatory high-resolution central schemes for conservation laws. The first version is based on a dimension-splitting concept, while the second on is based on a detailed computation on cell average in smooth and singular regions of a controlled volume. In both cases, numerical simulation are performed and show high-resolution.

Critical Thresholds Phenomena in a class of Euler-Poisson Equations of Fluid Dynamics and Plasma Physics

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Abstract

This talk is concerned with the critical thresholds phenomena in a class of Euler-Poisson equations of fluid dynamics and Plasma Physics. We show that for each set of equations there exists a *critical threshold* such that if the initial data exceed the critical threshold, then the solution of the equations behaves in one way whereas if it does not exceed the threshold the solution behaves in a qualitatively different fashion. We also describe the long time behavior of the solutions of some equations related to the evolution of pressureless gases of charged and uncharged particles with and without the effects of viscosity as well as the relaxation.

This talk reflects collaborative work with Shlomo Engelberg (Jerusalem, Israel) and Eitan Tadmor(UCLA).

Nonlinear Schroedinger Equations and Conservation Laws

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Abstract

The focusing cubic nonlinear Schroedinger equation (NLS) on the real line has attracted much attention as a completely integrable system. Recently Terng and Uhlenbeck used it to study the related geometric Schroedinger equation (GSE), also called Schroedinger flow in the work of Ding and YD Wang, and established global existence results. Another related result is that of Bourgain's on the well-posedness of the NLS on the circle. In recent joint work with HY Wang and YD Wang, via a geometric analytic approach, we established global existence for GSE as well as NLS on the real line as well as the circle and with values in Hermitian locally symmetric spaces (for GSE) and Lie algebras (for NLS). At the heart of our argument is a conservation law which makes use of geometric properties of Hermitian locally symmetric spaces. In this talk, we will focus on this conservation law and discuss its manifestation in the GSE as well as NLS settings.

Finite Volume WENO Schemes and the Treatment of Negative Weights

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Abstract

We present two versions of high order finite volume WENO (weighted essentially non-oscillatory) schemes for solving two dimensional conservation laws on rectangular meshes and also on arbitrary triangular meshes. A strategy in handling negative linear weights is discussed and also extensively tested numerically. This strategy can be used not only for high order WENO schemes for two dimensional conservation laws where negative linear weights always appear, but also for other applications such as a WENO approximation to second derivatives where negative linear weights appear. Numerical results of scalar and systems of two dimensional conservation laws will be shown. This work is joint with Jing Shi and Changqing Hu of Brown University.

Moving Mesh Method based on Harmonic Maps and its Application to Hyperbolic Conservation Laws

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Abstract

Adaptive mesh methods have been widely used in the last decade for solving partial differential equations that involve large solution variations, such as shock waves, boundary layers or wave fronts. In order to resolve such nonuniformities using a minimum number of mesh points, it is desirable to concentrate the mesh within the areas of large solution variation. In practice, there are three type of adaptive methods using finite element approach, namely h-method (mesh refinement), p-method (order enrichment) and r-method (mesh motion). The r-method, which will be discussed in this talk, is also known as moving mesh method. In the h-method, the overall method contains two parts, a solution algorithm and a mesh selection algorithm. In principle, these two parts are independent of each other in the sense that the change of the PDEs will affect the first part only. However, in most of the existing moving mesh methods, these two parts are strongly associated with each other such that the change of the PDEs will result in the rewritting of the whole code. In this talk, we will use a moving finite element scheme which consists of two *independent* parts: a mesh-redistribution algorithm and a solution algorithm. The mesh-redistribution algorithm is independent of the solution algorithm which can be any of the standard codes for the given PDEs. In other words, the change of the PDEs will not affect the mesh-redistribution algorithm.

The moving mesh methods to be used in this research will be based on *harmonic mapping*. In order to obtain the harmonic maps efficiently, we construct the harmonic map between the physical mesh and the logical mesh by an *iteration* procedure. Each iteration step is to move the mesh *closer* to the harmonic map.

Our recent study has extended the moving mesh methods to solve variational inequalities and optimal control problems. Finite element schemes loose accuracy when approximating variational inequalities due to the existence of free boundaries. In our work, computational meshes are constructed by combining harmonic mapping and sharper *a posteriori* error estimators which are normally used in h-version adaptive finite element approximation. These estimators depend solely on the discrete solution and data and impose no constraints between consecutive time steps. Selecting appropriate error monitors is one of the most important issues that have to be addressed in implementing the moving mesh methods. It seems that the commonly used gradient based monitors are not suitable in solving the variational inequalities and optimal control problems. In this talk, we will show that the monitors based on equivalent *a posteriori* error estimates can handle the free boundaries very well.

In this talk, some applications on moving mesh methods to hyperbolic conservation laws will be discussed. Special attention will be paid to hyperbolic systems with stiff source terms.

This research is jointly with Li Ruo and Zhang Pingwen of Peking University, Liu Wenbin of the University of Kent, and Ma Heping of Shanghai University. Some preprints in this talk can be found in http://www.math.hkbu.edu.hk/~ttang/spec_adap.html

Energy and Helicity Preserving Schemes for Hydrodynamic and Magnetohydrodynamic Flows with Coordinate Symmetry

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Abstract

We introduce a new class of numerical scheme for incompressible Hydrodynamic and Magnetohydrodynamic flows with coordinate symmetry such as pipe flows, axisymmetric flows and flows on a sphere. In this case, we can define a generalized stream vector aligned with the symmetry direction and the flow is effectively two dimensional. We then give a general recipe to treat the nonlinear terms (convection term and the geometric source term) in such a way that both energy and helicity are conserved numerically (energy, magnetic helicity and cross helicity in the case of MHD equation). As a result, the scheme is nonlinearly stable and free from excess numerical viscosity. A second order scheme based on central differencing is illustrated as an example. The scheme can be easily generalized to higher order finite difference, finite element and spectral methods in this formulation. Moreover, mesh refinement near the boundary can also be built into the equation without extra cost.

For the general 3D MHD system, we developed a class of MAC-Yee scheme preserving both energy and cross helicity (we are not able to preserve the magnetic helicity for the general 3D MHD flow). We expect this class of methods will have many applications in fluid and plasma flow simulations, particularly in large time direct numerical simulation and numerical search for possible singularity formation.

Convergence theory for relaxation schemes

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Abstract

In 1995 Jin and Xin introduced the concept of relaxation schemes for conservation laws. A system of conservation laws may be embedded in a relaxation system with twice the number of dependent variables. The advantage is that the strongly coupled quasilinear system is transformed into a weakly coupled semilinear system with a simple structure of the nonlinearities in the source terms. The idea is now to discretize this relaxation system e.g. by a simple upwind discretization for the linear part.The hope is that the simplicity of the solver offsets increase in size of the system. Further, the method may be applied to problems where the calculation of flux Jacobians is expensive or even impossible.The talk will present results on the convergence and convergence rates to solutions of conservation laws for such schemes.

The presented results are joint work with Hailiang Liu (UCLA, USA) and Jinghua Wang (Academia Sinica, Beijing).

Gas-kinetic Schemes for Flow Simulations

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Abstract

In the talk, the gas-kinetic schemes for the compressible Euler and Navier-Stokes simulation will be introduced. Numerical properties of the kinetic schemes, such as the positivity and entropy conditions, will be analyzed. Also, the relations among the artificial dissipation, Godunov, and the kinetic schemes will be pointed out.

Quasi-least-squares mixed finite element scheme based on a discrete minus one inner project for steady flow with heat transfer

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Abstract

Fluid flow and heat transfer in rotating systems are not only of considerable theoretical interest, but also of practical importance in many engineering applications. Examples are the flow and heat transfer in compressor impellers, centrifuges, rotating heat exchangers, cooling channels of rotating machinery and particle separation devices. The system rotation, in conjunction with heating or cooling, introduces the centrifugal force, the Coriolis force and the Boyancy force in the momentum equations, which describe the relative motion of fluids with respect to an inertial frame of reference. Such body forces may induce a secondary flow in a plane perpendicular to the streamwise main flow. This could significantly affect the resistance to the fluid flow and convective heat transfer. As well, these forces may either enhance or impede each other in a nonlinear manner depending on directions of rotation and heat flux. This could results in a complicated bifurcation structure of the solution and the flow. Therefore, the development of effective numerical schemes plays a critical role in capturing such a complicated structure. We develop, in the present work, two quasi-least-squares finite element schemes and perform an analysis of existence and convergence for them.

Consider a non-inertial frame of reference with an angular rotating velocity ω. Let Ω be a bounded domain in R^d, $d = 2$ or 3, with the smooth boundary Γ or a bounded convex domain with the piecewise smooth boundary Γ. By using the useful Boussinesq approximation to deal with the density variation, the Navier-Stokes, continuity and energy equations governing the steady flow and heat transfer are given as follows:

$$\begin{aligned}
&(a) \quad -\nu\triangle\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + 2\omega\times\mathbf{u} + \nabla p = \beta(T - T_0)\nabla\phi, \quad \text{in } \Omega;\\
&(b) \quad \nabla\cdot\mathbf{u} = 0, \quad \text{in } \Omega;\\
&(c) \quad -\triangle T + \lambda\mathbf{u}\cdot\nabla T = g, \quad \text{in } \Omega;\\
&(d) \quad \mathbf{u} = 0, \quad T = 0, \quad \text{on } \Gamma;
\end{aligned} \tag{1}$$

where

$$\nabla\phi = \omega\times(\omega\times\mathbf{r}) + \mathbf{f}_0, \tag{2}$$

T and $\mathbf{u} = (u_1, \cdots, u_d)^\top$ are the temperature and the velocity respectively, ν the kinematic viscosity of fluid, λ rotational Groshoff number, β the coefficient of thermal expansion, $\mathbf{r}$ the position vector. T_0 is the reference temperature, $\mathbf{f}_0$ is the transitional acceleration of the origin of the non-inertial frame. p is a pseudo-pressure which absorbs any force residual implied by using the Boussinesq approximation.

The standard least-squares finite element scheme (LSFES) has been developed since the 1970's to approximate the solution of boundary value problems of linear partial differential equations (PDE). The associated least-squares energy functional is defined to be residuals of the PDE system by L^2-norm. The resulting linear algebraic system is symmetric and positive definite, whose solutions can be obtained by efficient algorithms such as multigrid methods and conjugate gradient methods.

There are, in general, two branches of analyses for problems under consideration. One is based on the Agmon-Douglis-Nirenberg's H^1-ellipticity theory for the first-order system with L^2 inner product. Another comes from H^{-1}-ellipticity theory and H^{-1} inner product. Based on the H^1 and H^{-1} theories, we can have continuous piecewise polynomial approximation with optimal rates of convergence in the H^1 and L^2 norms. Other methods based on the theory of 2m-order partial differential equations and the first-order systems have also been developed..

During the last decade, LSFES has been extensively applied to approximate numerically viscous incompressible flow. Recently, the least-squares methodologies are extended to nonlinear Navier-Stokes equations. Based on the standard least-squares principle in the L^2-norm and H^{-1}-norm, Bochev *et. al.* extended the velocity-flux methodology and velocity-vorticity-pressure methodology to solve the velocity-flux formulation of Navier-Stokes equations. They also developed some convergence theories by the canonical abstract approximation theory for the nonsingular solution branch of nonlinear problems. The nonlinear term in Navier-Stokes equations generates, however, some new nonlinear terms and asymmetry in the standard least-squares finite element schemes.

The motivation of the present work comes from the desire of developing one quasi-least-squares finite element schemes based on the discrete H^{-1} inner product to solve steady flow and heat transfer with system rotation. By the quasi-least-squares finite element scheme we mean a numerical scheme coming from the least-squares finite element principle and leading to, for nonlinear problems, a symmetric resulting algebraic system with no additional nonlinear terms. This scheme reduces to the LSFES for linear problems. Furthermore, the quasi-least-squares finite element schemes defined in the present work are globally convergent. Therefore, we may obtain solutions of nonlinear problems by the iteration starting from any initial approximation in a large scale of function set. This is in contrast to the standard least-squares finite element scheme, which is locally convergent near the solution.

While the canonical abstract approximation theory can be used to analyze the convergence of nonsingular solution branch of nonlinear problems, a new technique is developed to establish the convergence of various solutions including nonsingular and singular solutions. The quasi-least-squares finite element scheme is shown to be capable of capturing bifurcation-driven multiple solutions of nonlinear problems. This is important because the bifurcation-driven multiplicity is of great significance for practical applications. This new analyzing technique of convergence can also be extended to more complicated nonlinear problems, such as flows and heat transfer involving non-Newtonian fluids and fluids with a velocity-dependent or temperature-dependent viscosity.

Some Recent Results on Compressible Flow with Vacuum

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Abstract

In this talk, we will survey some recent results on the study of the viscous and invisid compressible flow with vacuum. It is well-known that the study on vacuum has significance in the investigation on some important physical phenomena. However, most of the important problems on vacuum are still open due to the singularities caused by vacuum.

Wave Interactions in Nonlinear Elastic Strings

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Abstract

We study a system modeling the dynamics of a nonlinear elastic string. This is a 6x6 system of hyperbolic conservation laws, which degenerates in a number of ways. We describe the nonclassical waves and examine their stability under Glimm interactions. We solve the Riemann Problem and give a detailed analysis of elementary wave interactions. We discuss Glimm's theorem, and describe features of the system when the total variation is large. There are complicated wave patterns, including infinitely many interactions in finite time, and both three- and four-resonances may be present. We discuss a strategy for understanding the large-time asymptotics of the system.

Construction of Shock Profiles

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Abstract

We consider the existence and stability of viscous shock profiles through effective numerical approximations and detailed time-asymptotic pointwise estimates.